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Date 11/21/86

Project No. E-25-602

School/Lab ME

Includes Subproject No.(s) G-37-628/Math

Project Director(s) Dr. Prateen V. Desai

GTRC / GIT

Sponsor National Science Foundation

Title Collaborative Research: Computer Aided Design for Castings

Effective Completion Date: 9/30/86 (Performance) 12/31/86 (Reports)

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E-25-602

RESEARCH PROGRESS

Progress Report on Task I: Design and Construction of a Geometric Modeler for Computer-Aided Design of Metal Castings

Summary of Progress:

The goal of Task I is to provide geometric models that can conveniently be linked to and used by the special software being developed in Task III for solidification simulation. Each geometric model will exist as a computer database, and Task I will provide software for convenient retrieval of information from these databases. The primary use of this information will be in generating finite element meshes for use in the Task III simulations. Mesh generation will be accomplished jointly by the Task I and Task III team members. Details of the procedure will not be clear until the simulation software is more complete.

Several recent accomplishments have prepared us for the work described above during the current year. They are described in the following narrative.

Installation of TIPS-1'77

The TIPS-1'77 geometric modeler [3] has been installed on an IBM 4341 system belonging to the College of Engineering at Georgia Tech. The software obtained was designed to drive a Tektronix 4010 graphics terminal, none of which is available in Mechanical Engineering. Hence, the subroutines RWIND, VSINI, VSTERM, DEVICE, and GPSLTM were replaced by subroutine GOURI, which uses IBM GSP software to drive IBM 3250 (black and white) and IBM 3279 (color) graphics terminals. Although hard copy of the graphics screens is not yet available, installation of an IBM 4250 graphics printer is expected in the near future.

The installed version of TIPS-'77 has the following capabilities:

- interactive solid geometric modeling using constructive solid geometry (CSG),

- sectioning of modeled objects,
- generation of a database representing the intersection of a modeled object with a grid of three sets of uniformly spaced planes, each set being normal to one of three global Cartesian axes,
- approximate calculation of "mass" properties,
- semi-automatic enmeshment of plane sections, and
- various rendering modes.

Although programs that interact with the TIPS-1'77 database to perform stress analyses and other functions are available from Cornell University, none of these has yet been acquired by Georgia Tech.

The installed version of TIPS-1'77 provides no topological information, although topology of the modeled object is inherent in the model.

Boundary data are available in TIPS-1'77 only through the model/plane grid intersection database referred to above. In principle, the plane grid could be made dense enough so that boundary information would be as accurate as desired. To capture boundary features whose scale is much smaller than that of the entire modeled object would, however, be very expensive by this technique.

Figures I-1 and I-2 show examples of TIPS-1'77 renderings. Each curve is the intersection of the model with one of the grid planes referred to previously.

Installation of PADL-2

The PADL-2 geometric modeler [4] is installed on a DEC VAX 11/750 computer belonging to the School of Mechanical Engineering. Although the software could be used with any of several graphics display devices, the only one of these presently available to us is the Tektronix 4010, as emulated by the Intecolor 2400. The School owns an HP-7221C plotter, which eventually will be used to make hard copies of PADL-2 renderings. This capability awaits the School's acquisition of appropriate graphics software by means of which the VAX could drive the plotter.

The installed version of PADL-2 supports the following activities:

. interactive solid geometric modeling using CSG,

. sectioning of modeled objects,

. generation of a database giving an accurate representation of the boundary of a modeled object, and

. various types of rendering.

Of the rendering modes supported by PADL-2, only the simplest (black and white, wireframe) is available on the Tektronix 4010. Because shaded color images cannot be rendered, much of the power of PADL-2 to communicate is presently unusable. Development of a software interface between the PADL-2 shaded image processor and the Tektronix 4027, a color device that can be emulated by the Intecolor 2400, is underway. However, because the Tektronix 4027 is not as powerful as other color devices, e.g., Lexida, supported by PADL-2, even the successful completion of this effort may not yield acceptable renderings.

Like TIPS-1'77, PADL-2 does not provide topological information about a modeled object directly to the user, but could, in theory, do so because the topology is inherent in the model.

Accurate boundary data can be extracted directly from PADL-2 because the system contains software specifically designed to produce an accurate boundary representation.

Graphics Display Hardware

As a result of a grant from the National Science Foundation and matching funds from the State of Georgia, the School of Mechanical Engineering has recently purchased a PS-300 graphics system manufactured by the Evans and Sutherland Corporation of Salt Lake City, Utah. This equipment allows the user to

- perform real-time rotation, translation, and scaling using manually-operated control dials,
- interact directly with computer via a cross-hair controlled by a stylus and pressure-sensitive pad,
- display color images with many thousand vectors, and
- perform quasi-dynamic hidden line removal.

Installation and testing of this equipment has only just begun. Eventually, the system will be used to display not only renderings produced by one or both of our geometric modelers, but also the 3-D meshes we plan to generate.

Summary of Proposed Work

The following activities will be pursued during the coming year:

- choosing between TIPS-1'77 and PADL-2 for further work,
- establishing the PS-300 as the display device for the chosen modeler,
- designing the enmeshment software to serve as interface between the chosen modeler and the software being developed under Task III.

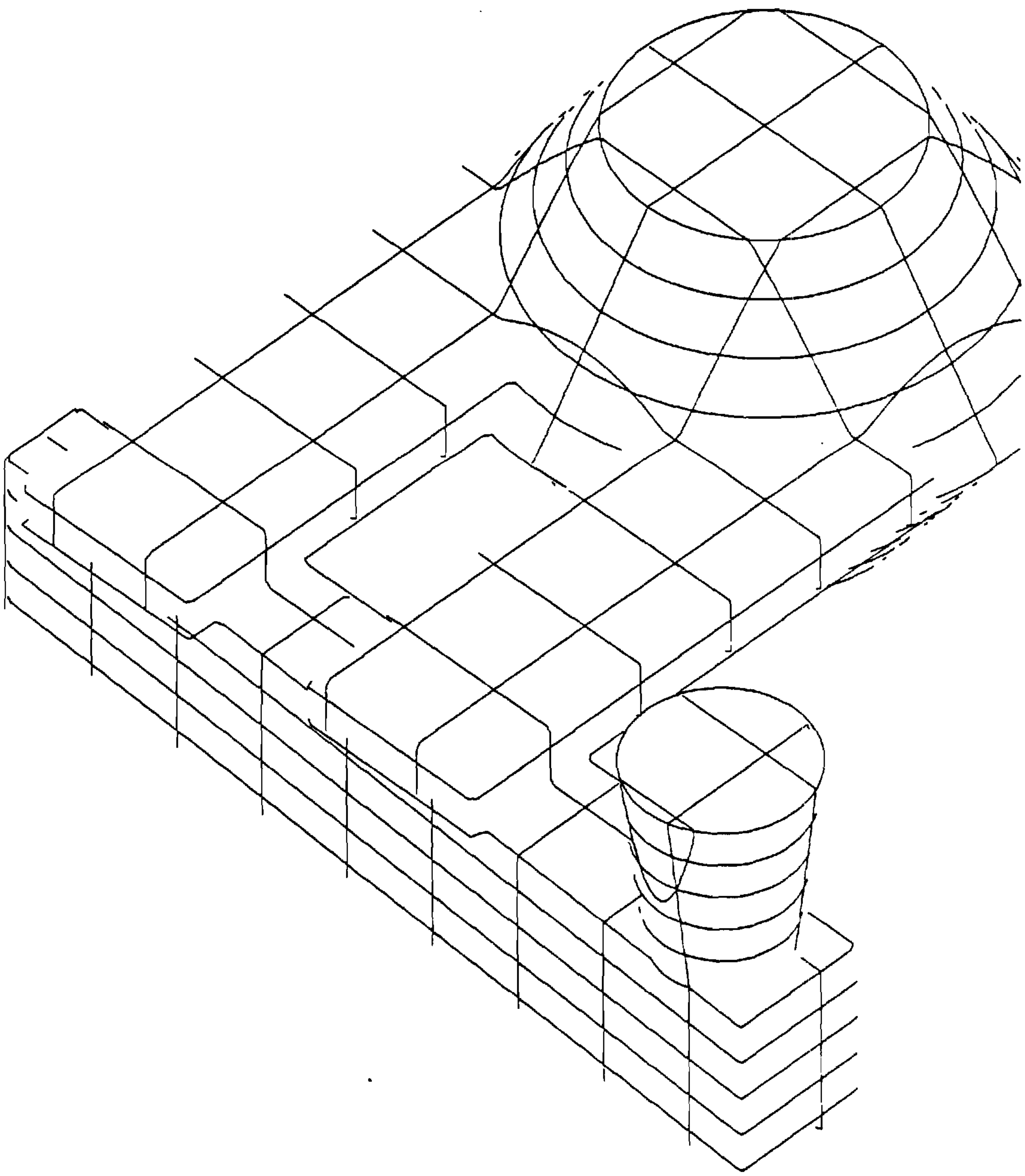

Figure I-1. A Casting and Gating System Modeled with TIPS-1'77.

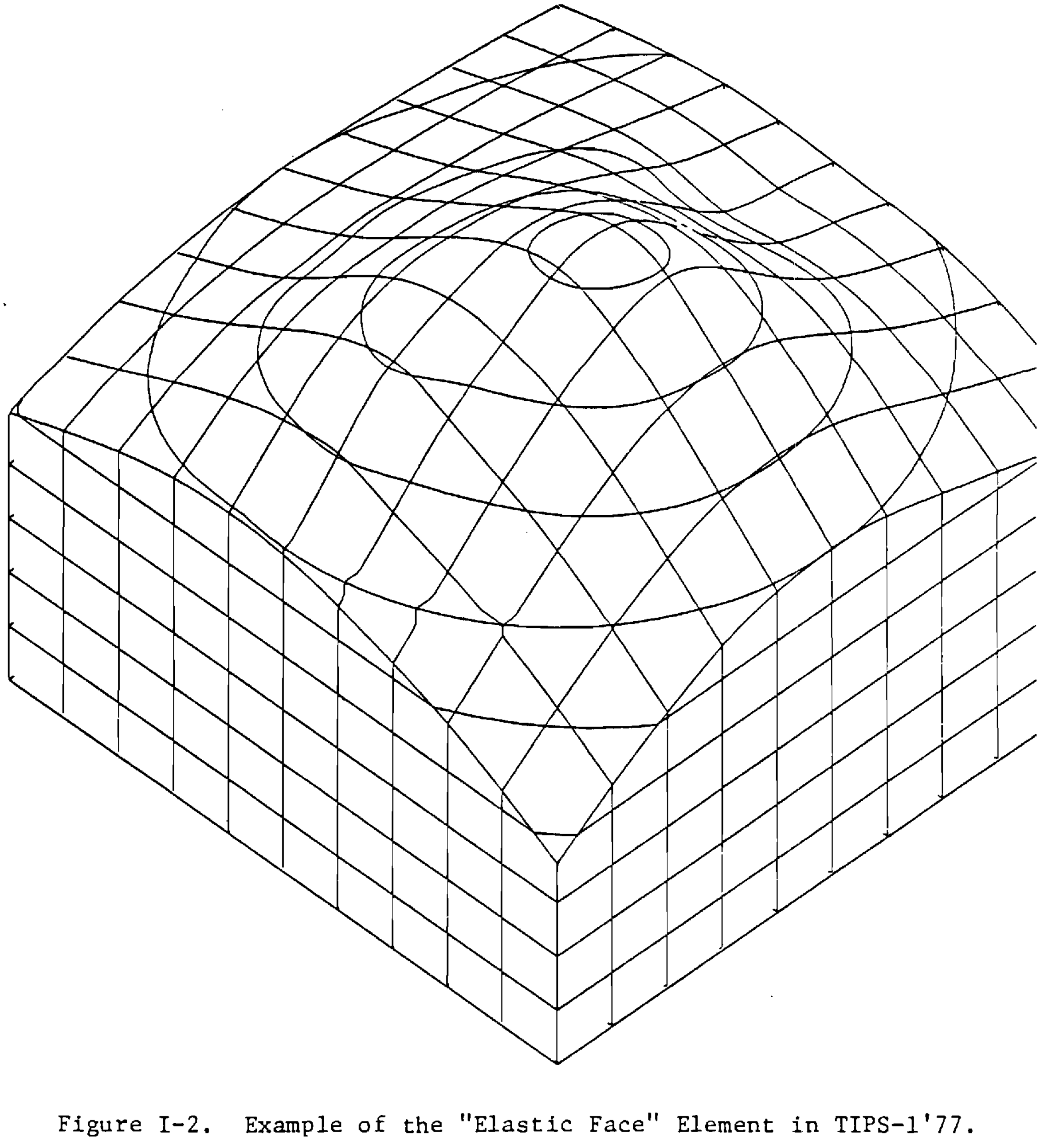

Figure I-2. Example of the "Elastic Face" Element in TIPS-1'77.

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Progress Report on Task II: Characterization of Transient Effects in Current and Future Molding Media

Summary of Progress

The two areas of primary concern in Task II are the development of a predictive model for the thermal conductivity of bonded molding sands and the analysis of the heat and moisture transport in green sand molds. The predictive model is considered essential because determination of the influence of sand type, initial moisture content, binder content, ramming density and temperature on the effective thermal conductivity of bonded sands by experimental methods alone is essentially an insurmountable task. The analysis of the transport mechanisms in green sands is important because the current state of knowledge is insufficient to predict or describe the increased chilling power observed when moisture is present in mold materials.

A new two-component thermal conductivity model, called the cylinder-element model, has been developed under this grant to predict the thermal conductivity of fluid-saturated sands. As the geometrical characteristics of sands can be described statistically but not analytically, the influence of particle size, size distribution and shape are determined empirically for use with the cylinder-element model.

The first phase of the experimental program used to complete and to verify the cylinder-element model has been accomplished. This effort included thermal conductivity measurements on a silica sand with round particles (Ottawa Silica) and an angular silica sand (Masonry sand) at various densities and with several saturating fluids having vastly different thermal conductivities (air, water, transformer oil and ethylene glycol). These measurements were used to establish two geometrical parameters which describe the influence of particle size and shape in the cylinder-element model.

Bonded sands are actually three-component systems containing sand particles, air and a bonding material such as resin or bentonite clay. The most recent modeling work in Task II included extending the applicability of the two-component model to three-component systems so that the thermal conductivity of bonded sands can be predicted. However, the thermal conductivity of the bonding material must also be known. Measuring the thermal conductivity of a continuous bonding media such as resin is not difficult, but bentonite bonds form porous structures at and near the sand particle contact points. Thus, methods must be developed for determining the effective thermal conductivity of the porous bentonite.

The results obtained from the model development phase of Task II and the associated experimental work can be summarized as follows:

1. The cylinder-element model is capable of predicting the thermal conductivity of two-component systems within about two percent of values determined experimentally.
2. The requisite geometrical parameters for a sand for use in the cylinder-element model for two-component systems can be determined from three experimental thermal conductivity measurements.
3. The cylinder-element model can also be used to evaluate the thermal conductivity of solid (sand) particles when this value is unknown.
4. The cylinder-element model has been extended for use with three-component systems, such as bentonite-bonded sands (bentonite-sand-air).
5. Experimental techniques have been developed for measuring the density of bentonite particles and the density of

dried bentonite as it exists in the bond between sand particles.

6. Thermal conductivity measurements have also been performed on dense samples of pure bentonite.

7. The design of the high-temperature probe used to measure the apparent thermal conductivity of bonded sands at elevated temperatures has been refined, and high-temperature measurements have been performed on bonded silica sands up to about $800^{o}C$.

The analysis of heat transport in green sands has also continued during this phase of work on Task II. An analytical model, based upon the conservation of mass, energy and momentum, has been further refined and will be used to predict the temperature, moisture and pressure distributions in a green-sand mold during the solidification of castings. The development of a finite difference program based upon this model was initiated during this phase of Task II and will be completed during the next phase.

Summary of Proposed Work

During the next phase of Task II, the development of the predictive model for thermal conductivity will continue and experimental verification of the model will be completed. Three-component systems whose binder is bentonite clay will be examined in detail. Experiments are planned for specimens of silica, zircon, chromite and olivine sands bonded with bentonite. Various combinations of binder content, initial moisture content and density which fall within the range of values commonly encountered in foundry practice will be used, and measured thermal conductivities will be compared with model predictions.

Implementation of the recently developed model of heat and mass transport in green sands in a finite difference computer program will be completed during the next phase of Task II, and theoretical models for the transport coefficients which appear in the formulation are to be developed. An instrumented test section compacted with green sand will be constructed, and experimental measurements of temperature and drying front location will be used to evaluate the accuracy of the analytical model.

Progress Report on Task IV: Thermal Convection in Gating Systems

Summary of Progress

The research under this task has centered on developing a predictive computational model to calculate transient forced convection heat transfer during the liquid metal passage through the sprue and the rest of the runner system. Previous work relating to this subject concerned a moving thermal contact heat transfer which was outlined in an earlier report [1]. Furthermore, some preliminary results obtained by using a new deforming finite element methodology were compared very favorably with those using the fixed finite elements. It was suggested to apply this technique to study the moving free surface, moving thermal contact problem in order to calculate heat loss from an advancing liquid metal front during its progress through an initially cold empty sand channel.

The moving free surface heat transfer studies have now been completed for a channel geometry. It has been shown [2] that the thermal energy equation for a fluid moving at a local velocity, $\underset{\sim}{u}\,(x,y,z,t)$, with reference to a coordinate frame deforming at a known rate, $\underset{\sim}{u}^{*}(x,y,z,t)$, may be written as

$$\rho c_p \left[\frac{\partial T}{\partial t} + (\underset{\sim}{u} - u^{*}) \bullet \underset{\sim}{\nabla} T \right] = k\nabla^2 T \qquad (1)$$

With reference to a material (Lagrangian) coordinate frame moving at the local fluid velocity, $\underset{\sim}{u}$, Equation (1) reduces to the usual Lagrangian form and for a fixed control volume, $\underset{\sim}{u}^{*} = 0$, it reduces to the usual Eulerian form of the thermal energy equation.

Comparison of Equation (1) with its conventional Eulerian form indicates that these two equations are formally identical when the convective term, $(\underset{\sim}{u} \bullet \underset{\sim}{\nabla}T)$, is replaced by the modified convective term, $[(\underset{\sim}{u} - \underset{\sim}{u}^*) \bullet \underset{\sim}{\nabla}T]$. In other words, the transformation of the conservation equations from an Eulerian frame (ef) to that for the deforming coordinate frame (df) only requires a modification in the convective term such that for any function $\psi(t)$

$$(\underset{\sim}{u} \bullet \underset{\sim}{\nabla}\psi)_{ef} \longrightarrow [(\underset{\sim}{u} - \underset{\sim}{u}^*) \bullet \underset{\sim}{\nabla}\psi]_{df} . \qquad (2)$$

Therefore, the transformation is of the form

$$\underset{\sim}{u}_{fluid,df} = \underset{\sim}{u}_{fluid,ef} - \underset{\sim}{u}^*$$

Then in a domain R_1 where the Eulerian fluid velocity is $\underset{\sim}{U}$, and in a solid domain R_2 where the fluid velocity is zero, the respective expressions become

$$\underset{\sim}{u}_{fluid,df} = \underset{\sim}{U} - \underset{\sim}{u}^* \quad ; \quad \underset{\sim}{u}_{fluid,df} = 0 - \underset{\sim}{u}^* = -\underset{\sim}{u}^*$$

Equation (1), written in the deforming coordinate frame, must be integrated over the (arbitrary) control volume, $V^*(t)$, whose velocity is $\underset{\sim}{u}^*$ (x y,z,t).

In the finite element matrix formulation for a two dimensional domain the arbitrary control volume, $V^*(t)$, is replaced by a two-dimensional finite element subdomain, $\Omega^e(t)$. Within this subdomain, spatial coordinate $X(x_i,t)$ is expressed by a series of products of the time-independent shape functions, $N_j(x_i)$ and the time-dependent nodal coordinates, $a_j(t)$; i.e.,

$$X \simeq \sum_{j=1}^{p} N_j(x_i)a_j(t) \qquad (i = 1,2) \quad . \qquad (3)$$

Similarly, a trial function, $\hat{T}$, which is an approximation of the unknown function, T , is expressed by

$$T \simeq \hat{T} = \sum_{j=1}^{p} N_j(x_i)T_j(t) \qquad (i = 1,2) \quad . \tag{4}$$

Therefore, the conventional Galerkin procedure may be applied to Equation (1) to yield

$$[C]\{\dot{\hat{T}}\} + [K]\{\hat{T}\} = 0 \quad , \tag{5a}$$

where components of [C] and [K] are written as

$$C_{ij} = \iint_{\Omega^e} \rho c_p \; N_i N_j dxdy \tag{5b}$$

and

$$K_{ij} = \iint_{\Omega^e} N_i(\underset{\sim}{u}-\underset{\sim}{u}^*) \; \underset{\sim}{\nabla} N_j dxdy + \iint_{\Omega^e} k\left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) dxdy \tag{5c}$$

For brevity, the boundary conditions are not explicitly stated.

The solution of this formulation, applicable to the problem of thermal energy loss in a sand gating system, involves a liquid metal flow through a channel. The case examined for discussion purposes involves a liquid metal flow at 100 cm/sec through a channel of semi-width of 1 cm and a length of 20 cm in a nonmetallic medium. The problem domains are discretized by using a total of 494 four-node bilinear quadrilateral isoparametric elements (100 elements for the liquid region, 10 elements for the "fictitious" layer, and 384 elements for the solid region) as shown schematically in Figure IV-1.

A fictitious finite element interface layer [3], which obviates the early response errors and inaccuracies at the interface associated with conventional computational schemes for thermal contact between dissimilar materials, is implemented by introducing an interface layer of elements with very small thermal capacity and a very large thermal conductivity. The size of the layer, the computation time step and the thermal properties are chosen such that the thermal response of the fictitious layer becomes independent of the Fourier number of the layer The accuracy of the solution obtained by using a fictitious layer formulation is independent of the thermal conductivity ratio of the two materials [3].

The deformation of the mesh pattern includes an element group (I) (10 layers normal to the flow direction) that is linearly expanding and a group (II) (6 layers normal to the flow direction) which is translating without deformation.

The motion of the free surface is accommodated by varying the number of layers normal to the flow direction in the groups (I) and (II). At the beginning of the computation (when the free surface is about to enter the channel) the number of layers of group (I) is ten, and that of group (II) is six, as shown in Figure IV-2(i). As the free surface reaches 2 cm into the channel the number of layers of group (I) is reduced to eight and that of group (II) is increased to eight. This is shown in Figure IV-2(ii).

After the free surface reaches a distance of 10 cm from the channel entrance, only five layers are linearly expanding, as shown in Figure IV-2(iii). This deformation pattern is used to maintain reasonably small element sizes near the free surface region where severe temperature gradients are experienced. The material thermal properties are selected by considering liquid pure aluminum in region 1 and a typical casting sand in region 2. The heat transfer coefficient, h, for the free surface, which must include the radiation effects, is assumed to be 100 W/m^2K.

Two time step sizes (Δt = 0.0025 $\rightarrow$ 0.005 and Δt = 0.005 $\rightarrow$ 0.01 sec) were examined, with the temporal integration scheme changed from the fully implicit to the Modified Crank-Nicholson scheme when the free surface reaches 2 cm from the channel entrance. No significant changes in the results for these time step sizes were observed, and the larger time step ($\Delta t = 0.005 \rightarrow 0.01$ sec) was used subsequently. The interface temperature at the free surface, located at y_{fs} (or at time, t, after the free surface enters the channel), and the metal-mold interface temperature distributions from the channel entrance to the free surface (at t = 0.02, 0.1 and 0.2 sec) are shown in Figure IV-3. The temperature distributions at the free surface corresponding to these three times (Figure IV-4) show that the interface temperature at and near the free surface experiences a large drop. However, the following hot liquid raises this temperature.

In conclusion, the analysis of two-dimensional transient conjugate heat transfer with a moving free surface can be performed with a continuously deforming finite element method. For finite geometries, this problem is inadequately described in either a purely Lagrangian or a purely Eulerian frame. The appropriate formulation begins with the derivation of the conservation equations for a control volume deforming at a known rate. These equations become valid in a deforming coordinate frame under a transformed formulation employing the velocity of the fluid relative to that of the deforming coordinate frame.

The implementation of this formulation is straightfoward and uses conventional finite element techniques. Unlike other numerical solution techniques in which the moving boundary deformation is temporarily held stationary, this approach incorporates a continuous deformation. The free surface motion is modeled by a grid of deforming finite elements. Although the choice of the appropriate deformation pattern is very much problem dependent, it does not necessarily limit the application of the method.

This method can be applied to calculate the conjugate heat transfer as a hot liquid with a free surface advances into a cold channel. The temperature at the interface between the liquid and the channel wall during the filling transient drops significantly as the free surface is approached, and this drop is recovered immediately following the passage of the free surface. In the context of metal casting processes, this phenomenon is viewed as being responsible for the remelting of a solidified crust at the interface between a liquid metal and the walls of a gating system during initial filling.

Proposal for Future Work

The main aim of the future work will be to establish suitable design criteria for the thermal performance of two dimensional or axisymmetric gating sprues in sand casting systems. A complete solution of the heat transfer problem requires the determination of the temperature distribution in the mold wall and both the flow field and temperature distributions in the flowing metal. A prior calculation of the constant (with respect to time), non-uniform velocity field enables independent solution of the thermal energy equation. Thus the problem reduces to a transient conjugate heat transfer problem requiring a simultaneous calculation of the temperature fields in the mold and in the metal.

An analytical solution is very complex due to the conjugate nature of the problem with unknown time-dependent boundary conditions at the mold/metal interface. A numerical solution via the finite element method will be used to solve the problem.

Due to the low Prandtl number of liquid metals, a slug flow assumption may be made. The sprue may be assumed to be instantaneously filled since the main emphasis is on studying the heat transfer characteristics of gating sprues. A fixed finite element mesh describes the domains of interest. To smooth out the very high initial thermal gradients at the interface, a ficitious layer of elements having a low thermal capacity and a high thermal conductivity (in the direction of maximum heat flux) will be used.

The problem will be solved for several mold-metal combinations, horizontal runners and vertical down sprues, several taper angles of the sprue and for several inlet Peclet numbers. The thermal diffusivity ratio of the mold and the metal, the extent of taper and time (in the form of a Fourier number) are the important input parameters that will govern the solutions. The effects of these on the interface temperature will be studied and represented in the form of graphs usable by the practising foundry engineer.

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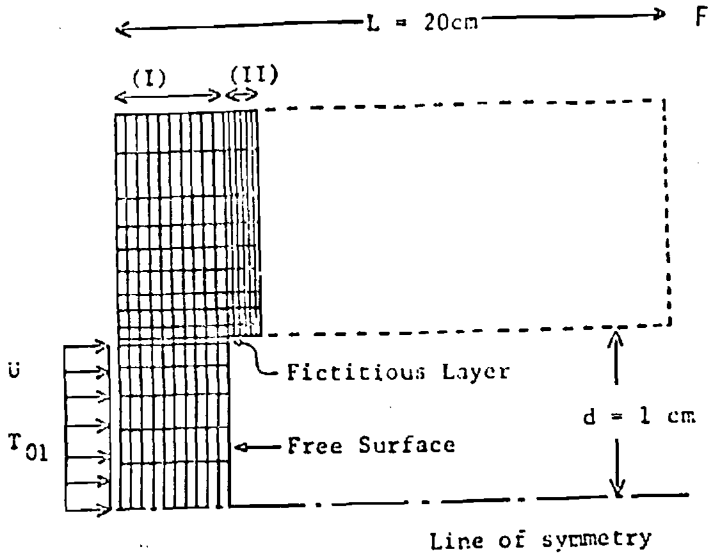
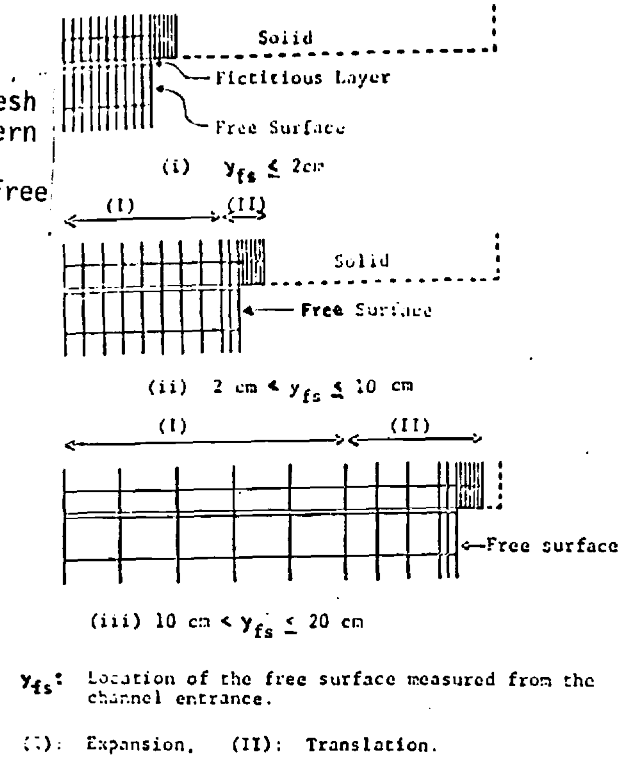


Figure IV-1. Finite Element Mesh System Representation During Early Stage of Free Surface Advance (y_{fs} = Free Surface Advance into Channel < 2 cm).

Figure IV-2. Representation Mesh Deformation Pattern Changes with the Advanced of the Free Surface.



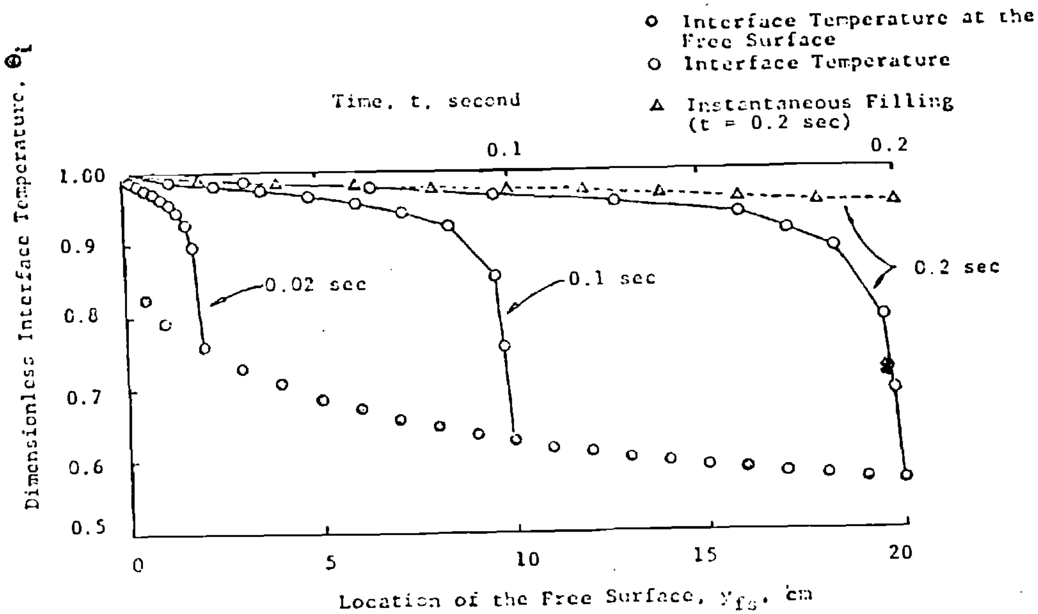


Figure IV-3. Response of the Interface Temperature at the Free Surface and Interface Temperature Distribution at Certain Times.

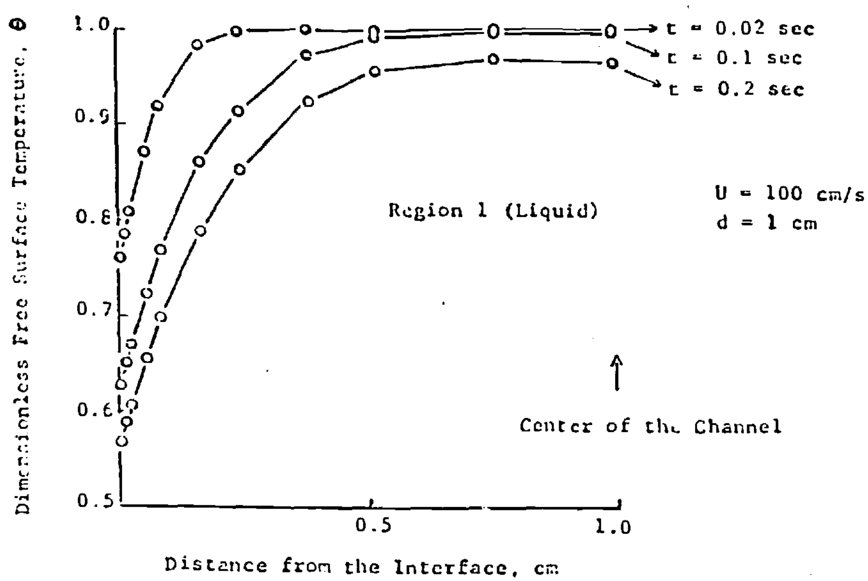


Figure IV-4. Temperature Distribution on the Free Surface at Various Times.

Progress Report on Task VII: The Control and Prescription of Heat Flux at the Casting-Mold Interface

Summary of Progress

The control and prescription of interfacial heat flux at the casting-mold boundaries has very wide implications. Not only does it have the potential to control the shape of solidification profiles to promote for example correct sequential solidification, but it has the promise of the enhancement of casting quality and casting machinability through control of cooling rate, which in turn regulates some of the many metallurgical parameters involved here.

The present directions of Task VII are pointed towards developing understanding of

(a) the natural variation of heat flux as a function of geometry, mold medium and elapsed time;

(b) the potential of devices such as the heat-pipe for regulating or otherwise enhancing the naturally occurring heat flux patterns in the mold.

In the first area, useful progress has been made [1] in examining two-dimensional corner-like features. Previously, a series of papers [2] described how a boundary heat-flux related function can be used to replace the many nodes required to simulate the body of the mold in finite difference or finite element computations of freezing front progress. The example being modeled currently, a cruciform [1], has special relevance to feeder head design in steel and ductile iron castings [3]. Figure VII-1 shows a two-dimensional mesh using linear, triangular elements which represent one-eighth of the geometry of the shape. The finite element method (FEM) model, developed to obtain flux data maps, represents a solution to the differential enthalpy equation first solved using the finite difference method (FDM) by Sarjant and Slack [4]. This technique, in the FDM form, has since been used extensively by P. Hansen of the Danish Technical University [5,6]. Prof. Hansen has had several stays at Georgia Tech as a visiting scientist and is expected to contribute to this aspect of our work during projected visits in 1985. Details of the recently developed FEM model have been described in detail in the current progress report [1]. Figure VII-2 shows some preliminary results obtained using the technique for a simple one-dimensional example, a constant thermal property system consisting of pure aluminum in contact with a silica sand mold. The FEM result compares well with that of the exact analytical solution which exists for this very simple geometry. Utilizing the temperature gradient information, the boundary heat flux can be calculated. The corresponding mold temperature profiles and the position of the freezing front are shown in Figure VII-3. Preliminary results for a two-dimensional external-internal corner feature, as enmeshed in Figure VII-1, are shown in Figure VII-4. In this figure the results are compared with those of our analytical predictions due to Wei [7].

Currently, further two-dimensional systems are being examined, for example, a long bar fed by an end-located riser. The molding medium is silica sand, the casting medium is aluminum A357, a long-freezing range alloy.

In the second area of endeavor, two master's theses have been presented which deal specifically with the simulation of heat pipes in contact with two-dimensional mold assemblies containing a molten alloy

[8,9]. The second study introduced the formation of an air gap at the mold metal interface. In this case the end of the heat-pipe itself was allowed to be in contact with the molten metal (a Pb-Sn) alloy. A direct comparison of the extent of the effects of the air gap was made with a system where the air gap was not present. Furthermore, a parallel comparative model study was made of a water-cooled chill in contact with a similar two-dimensional mold assembly. It was seen that the air gap drastically reduces the cooling power of both the heat-pipe and chill cooled systems. It must be pointed out, however, that there exist a variety of casting methods where such air gap effects can be minimized, for example squeeze casting. Recently published results of a simulation by El-Mahallawy and Taha [10] suggest that solidification times of corresponding casting locations may be halved by applying pressure during the solidification of a Pb-19.2% Sn alloy in a mild-steel die. They also reported on a concommitant refinement of dendrite arm spacing within the casting microstructure.

Experimental work will be mounted during the next year by the present team, utilizing a custom-designed heat pipe-cooled mold assembly. Preliminary work on a uni-directional solidification apparatus which incorporates a means of either casting against a chill or a heat pipe face using either the suction method of casting (chill-up) or the more standard (chill-down) approach has proceeded throughout the last year. A number of runs using both low melting point alloys and pure aluminum have been conducted and various design improvements suggested for the apparatus.

Summary of Proposed Work

It will be recalled that the original terms of reference of this task involved not only the use of heat pipes in controlling and prescribing heat-flux at the mold-metal interface but also the study of the natural heat-flux profiles that occur in moderately complex features of shaped castings. Since the features concerned contribute to the sequential aspects of solidification and thus casting quality through end and corner effects, it is important that these effects be understood.

Work will continue therefore on mapping the natural variation of heat flux occurring during and immediately after solidification using the FEM-based solution of the differential enthalpy equation. Special aspects which will be studied will be the effects of casting superheat and of solidification range.

Turning to the research associated with the use of heat pipes, work is currently being planned which will be concerned with model scale experiments involving a heat-pipe cooled enclosure containing high purity aluminum or a low melting point alloy. In view of the results of the two simulation studies which established that the formation of an air gap severely restricts the heat extractive capacity of a heat pipe, the effects of metallostatic head or system pressure will, of necessity, be included in this experimental phase of the work.

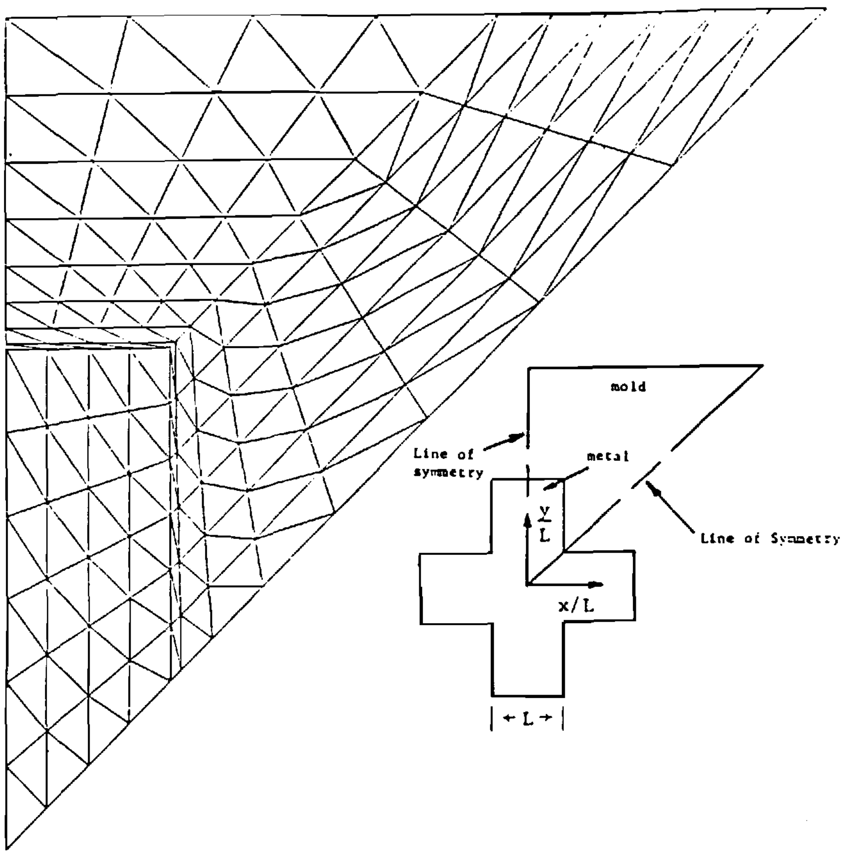


Figure VII-1. Mesh Used for the 2-D Cruciform Shape.

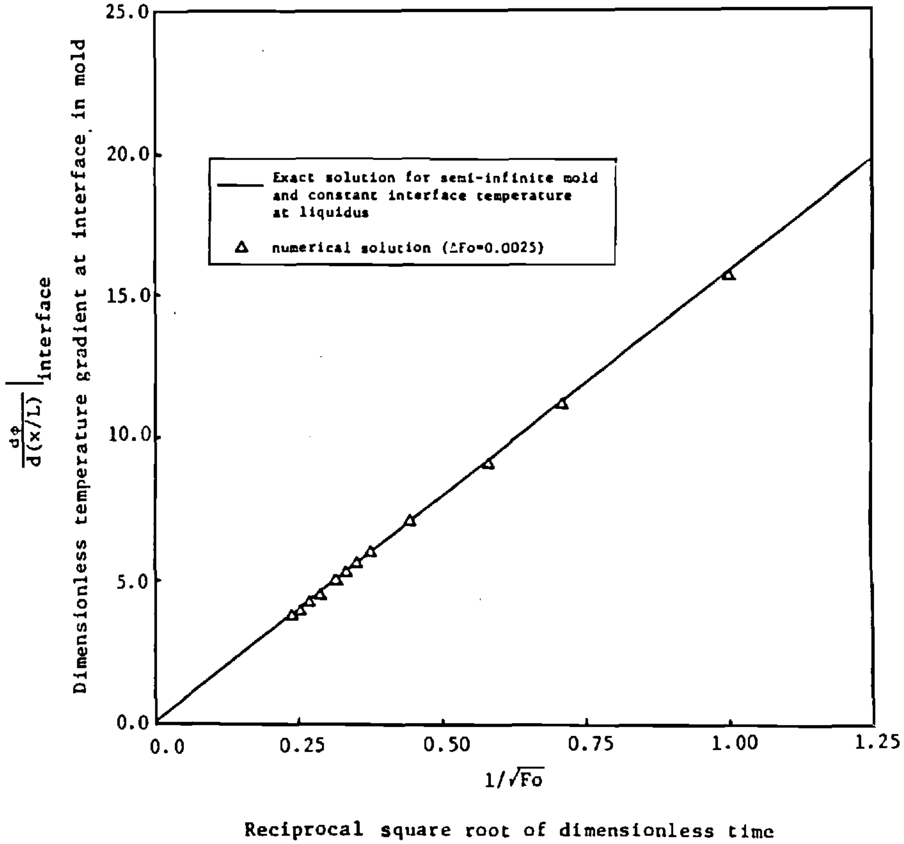


Figure VII-2. Numerically Calculated Dimensionless Temperature Gradients at the Interface as Compared to the Constant Interface Temperature Analytical Solution for the Aluminum/Silica Sand System ($Fo_m = \alpha_m Fo/\alpha$), Lumped Capacitance in Metal Only.

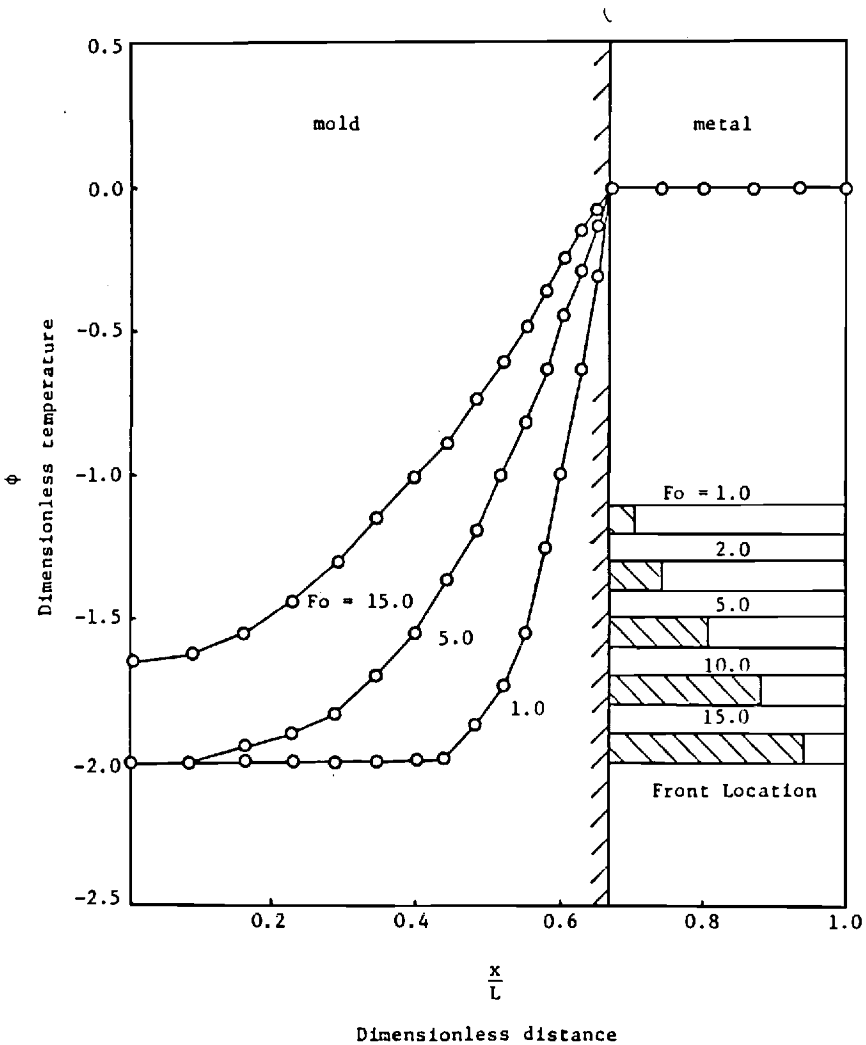


Figure VII-3. Dimensionless Temperature Profiles and Solidification Front Location at Various Values of Dimensionless Time, Lumped Capacitance in Metal Only.

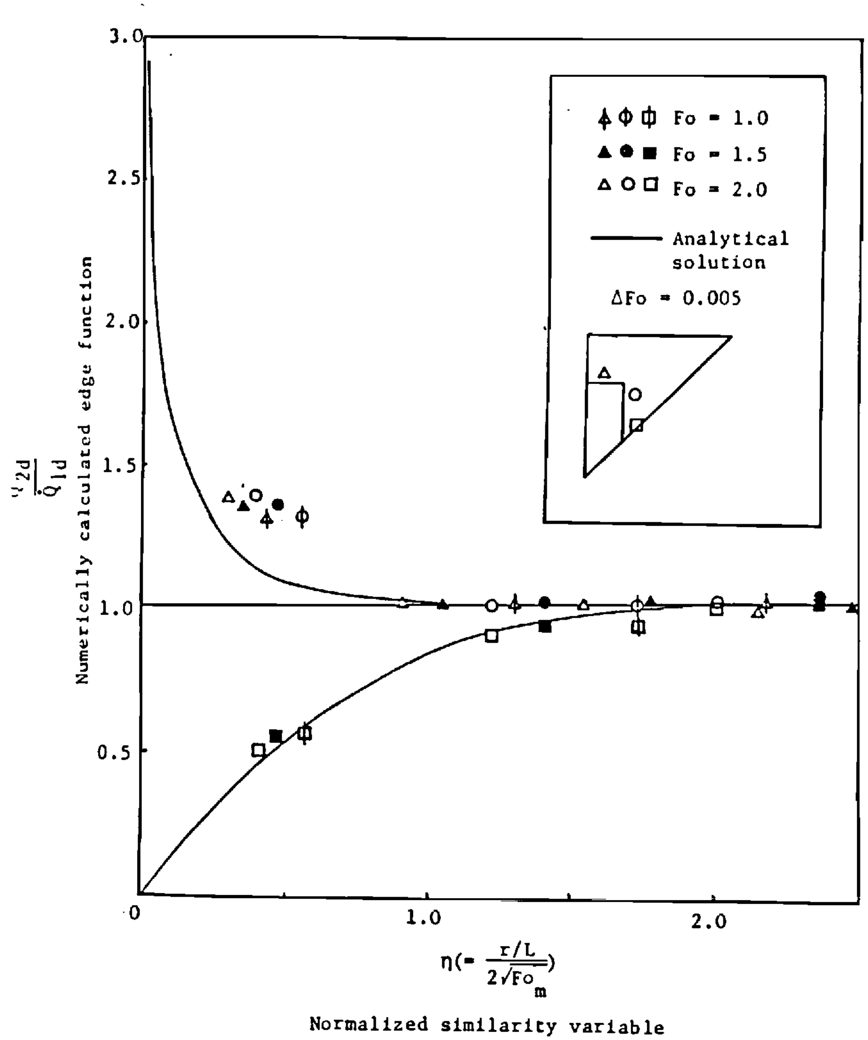


Figure VII-4. Numerically Calculated Values of the Edge Function $E(\theta_o) = \frac{\dot{Q}_{2d}}{\dot{Q}_{1d}}$ vs. $\eta(Fo_m = \alpha_m/\alpha Fo)$ Superimposed upon the Analytical Results of Wei [12].

REFERENCES

1. J. Moosbrugger and J. T. Berry, Task VII, Progress Report No. 1, NSF Grant MEA 82 11524 (November 1984).

2. See publications of Georgia Tech CADCAST Team, Numbers 10, 11, 14, 18, and 29, listed on pages 11-13 of the present report.

3. R. Sillen, Swedish Patent No. 8002279-1 B22 C 9/08.

4. R. J. Sarjant and M. R. Slack, "Internal Temperature Distribution in the Cooling and Reheating of Steel Ingots," J. Iron and Steel Inst., Vol. 177, pp. 428-444 (1954).

5. P. N. Hansen, "Solidification and Related Structure as a Function of Metal/Mold Boundary Temperature," Solidification Technology in the Foundry and Casthouse, Conf. Proc., The Metals Society, London (1980).

6. P. N. Hansen, "Numerical Simulations of the Solidification Process," Solidification and Casting of Metals, Conf. Proc., The Metals Society, London (1977).

7. C. S. Wei, "An Analysis of the Transient Corner Effect of Heat Conduction and Its Application to Casting Solidification," Ph.D. Dissertation, Georgia Institute of Technology (1982).

8. K. J. Wells, "Two-Dimensional Numerical Simulation of Casting Solidification with Heat Pipe Controlled Boundary Conditions," M.S. Thesis, Georgia Institute of Technology (1982).

9. A. Lodhia, "A Theoretical Study of Interfacial Resistance in Metal Casting with Heat Pipe and Chill," M.S. Thesis, Georgia Institute of Technology (1984).

10. M. El-Mahallawy and M. Taha, "Simulations and Experiments in Pressurized Solidification and Rapid Solidification," Modeling of Casting and Welding Processes II (Ed. J. A. Dantzig and J. T. Berry), AIME (1984), pp. 155-159.

PROGRAM OBJECTIVES FOR YEAR III

The objectives for the third year of the current project may be summarized as follows:

Task I

(i) Complete evaluation of TIPS-1'77 and PADL-2 modelers, consequently choosing one best suited to casting related activity.

(ii) Design enmeshment software to serve as interface between modeler chosen and simulation software developed under Task III at the University of Michigan.

Task II

(i) Continue development of predictive model for dry sand system thermal conductivity, together with experimental verification.

(ii) Implement recently developed model for heat and mass transport in green sands, together with experimental measurements.

Task III

(i) Establish design criteria for evaluating thermal performance of two-dimensional and axisymmetric gating systems.

(ii) Utilizing the numerical solution obtained for this transient conjugate heat transfer problem, examine a variety of system elements including runner bar and down sprue features for various mold/metal media combinations. The data base created will be in a form that is directly usable by the practising foundry engineer.

Task VII

(i) Continue the mapping of the natural variations of heat flux with common geometric features as they are affected by the mold medium and elapsed time.

(ii) Commence experimental work on using a heat pipe to control heat flux patterns, hopefully studying effects of metallostatic pressure within the system.

In addition to the above research, the team will continue its pattern of meeting annually with leaders in the metal casting industry during its yearly Advisory Board meetings. The team will also continue its contact with the various overseas research groups working in solidification simulation related topics.

SUMMARY OF ALL CURRENT AND PENDING RESEARCH SUPPORT [FROM WHATEVER SOURCE]

The following information should be provided for each investigator and other senior personnel (see p. 6). Failure to provide this information may delay consideration of the proposal.

	A	B	C	D	E[1]		F
	Source of Support[3]	Project Title[2]	Award Amount (or Annual Rate)	Period Covered By Award	Person-Months Or % of Effort Committed To The Project		Location Where Research Is/Will Be Performed
					ACAD.	SUMM.	
I. (Name of Principal Investigator) J. T. Berry							
A. *Current Support* List—If none, Report none	NSF_1	Δ	$454K	'83-'86	20%	20%	ATL.
B. *Proposals Pending* 1. List this proposal	Lockheed	*	24K	'84-'85	5%	25%	ATL.
2. Other pending proposals, including renewal applications. If none, report none.	See A	—	—	—	—	—	—
3. Proposals planned to be submitted in near future. If none, report none.	NSF_2	†	364K	'85-'86	40%	66%	ATL.
II. (Name of co-principal investigator and/or faculty associate)	—	—	—	—	—	—	—
A. NSF_1 - Desai, Hartley, Newcomer, Colwell, Meyers							
B. Lockheed - Berry							
C. NSF_2 - Desai, Hartley, Newcomer, Meyers							
III. Transfer of Support If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.	—	—	—	N/A	—	—	—
IV. (Other agencies to which this proposal has been/will be submitted)	—			None			

[1] Nonacademic researchers may report percentage of total research effort using the first column only.

[2] Entry of project title should be by number coding (i.e., 1,2,...) and the full titles should be identified according to number at the bottom of the form (i.e., 1. full title; 2. full title....)

[3] Include NSF, other Federal and State Agencies, and private sources.

ΔA Computer-Aided Design System for Castings (NSF_1)

*A Study of the Machinability of Abrasive Inclusion Containing Alloys

†A Computer-Aided Design System for Castings - Univ.-Ind. Cooperative Research Program

SUMMARY OF ALL CURRENT AND PENDING RESEARCH SUPPORT
[FROM WHATEVER SOURCE]

The following information should be provided for each investigator and other senior personnel (see p. 6). Failure to provide this information may delay consideration of the proposal.

	A	B	C	D	E[1]		F
	Source of Support[3]	Project Title[2]	Award Amount (or Annual Rate)	Period Covered By Award	Person-Months Or % of Effort Committed To The Project		Location Where Research Is/Will Be Performed
	G. T. Colwell				ACAD.	SUMM.	
I. (Name of Principal Investigator)	$NASA_1$	*	$ 50K	1/85-12/85	10%	10%	ATL.
A. *Current Support*	$NASA_2$	**	$111K	8/83-12/85	10%	10%	ATL.
List—If none, Report none	Air Force	***	$260K	9/83-9/85	20%	20%	ATL.
B. *Proposals Pending* 1. List this proposal	NSF	Δ	$454K	'83-'86	5%	10%	ATL.
2. Other pending proposals, including renewal applications. If none, report none.	—	—	—	—	—	—	—
3. Proposals planned to be submitted in near future. If none, report none.	—	—	—	—	—	—	—
II. (Name of co-principal investigator and/or faculty associate)	—	—	—	—	—	—	—
A. $NASA_1$ - Hartley							
B. $NASA_2$ - Hartley		D. NSF - Berry, Desai, Hartley, Newcomer, Meyers					
C. Air Force - Wepfer							
III. Transfer of Support If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.	—	—	—	N/A	—	—	—
IV. (Other agencies to which this proposal has been/will be submitted)	None						

[1] Nonacademic researchers may report percentage of total research effort using the first column only.

[2] Entry of project title should be by number coding (i.e., 1,2,...) and the full titles should be identified according to number at the bottom of the form (i.e., 1. full title; 2. full title....)

[3] Include NSF, other Federal and State Agencies, and private sources.

* Thermal Control Model for Space Station
** Modeling of Transient Heat Pipe Operation
*** Heat Pipe Cooling of Missile Sites
Δ A Computer-Aided Design System for Castings

SUMMARY OF ALL CURRENT AND PENDING RESEARCH SUPPORT [FROM WHATEVER SOURCE]

The following information should be provided for each investigator and other senior personnel (see p. 6). Failure to provide this information may delay consideration of the proposal.

	A	B	C	D	E[1]		F
	Source of Support[3]	Project Title[2]	Award Amount (or Annual Rate)	Period Covered By Award	Person-Months Or % of Effort Committed To The Project		Location Where Research Is/Will Be Performed
					ACAD.	SUMM.	
I. (Name of Principal Investigator) P. V. Desai							
A. *Current Support* List—If none, Report none	NASA K.S.C.	*	$40K	1/85-1/86	20%	50%	ATL.
	NSF_1	**	454K	'83-'86	20%	20%	ATL.
	Ga. Tech	***	11.5K	'84-'85	20%	--	ATL.
B. *Proposals Pending* 1. List this proposal	See A	—	—	—	—	—	—
2. Other pending proposals, including renewal applications. If none, report none.	NSF_2	+	364K	'85-'86	10%	10%	ATL.
3. Proposals planned to be submitted in near future. If none, report none.	—	—	—	—	—	—	—
II. (Name of co-principal investigator and/or faculty associate)	—	—	—	—	—	—	—
III. Transfer of Support If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.	—	—	—	N/A	—	—	—
IV. (Other agencies to which this proposal has been/will be submitted)	—						

A. NSF_1 - Berry, Hartley, Colwell, Newcomer, Meyers, Stallybrass
B. NSF_2 - Berry, Hartley, Newcomer, Meyers

[1] Nonacademic researchers may report percentage of total research effort using the first column only.
[2] Entry of project title should be by number coding (i.e., 1,2,...) and the full titles should be identified according to number at the bottom of the form (i.e., 1. full title; 2. full title....)
[3] Include NSF, other Federal and State Agencies, and private sources.

* Flow-induced Vibrations
** A Computer-Aided Design System for Castings (NSF_1)
*** Gas Dynamic Lasers (GT/E-funds)
\+ A Computer Aided Design System for Castings - University-Industry Cooperative Research Program

SUMMARY OF ALL CURRENT AND PENDING RESEARCH SUPPORT [FROM WHATEVER SOURCE]

The following information should be provided for each investigator and other senior personnel (see p. 6). Failure to provide this information may delay consideration of the proposal.

	A	B	C	D	E[1]		F
	Source of Support[3]	Project Title[2]	Award Amount (or Annual Rate)	Period Covered By Award	Person-Months Or % of Effort Committed To The Project		Location Where Research Is/Will Be Performed
J. G. Hartley					ACAD.	SUMM.	
I. (Name of Principal Investigator)	NSF	1	$454K	'83-'86	20%	20%	ATL.
A. *Current Support*	NASA	2	111K	7/84-1/86	15%	15%	ATL.
List—If none, Report none	NASA	3	50K	1/85-1/86	15%	15%	ATL.
B. *Proposals Pending*	See A-1						
1. List this proposal	NASA	4	64K	1/85-10/85	20%	20%	ATL.
2. Other pending proposals, including renewal applications. If none, report none.	NSF	5	41K	6/85-3/86	0%	0%	ATL.
	NSF	6	18K	6/85-3/86	0%	0%	ATL.
3. Proposals planned to be submitted in near future. If none, report none.	EPRI	7	244K	9/85-9/88	25%	25%	ATL.
	NSF	8	364K	'85-'86	5%	5%	ATL.

II. (Name of co-principal investigator and/or faculty associate)

A. NSF-1: Berry, Desai, Colwell, Meyers, et al.

B. NASA-2 and NASA-3: Colwell

C. NASA-4 and NSF-5: Larson, Shelton

D. NSF-8: Berry, Desai, Newcomer, Meyers

III. Transfer of Support
If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period. N/A

IV. (Other agencies to which this proposal has been/will be submitted) None

[1] Nonacademic researchers may report percentage of total research effort using the first column only.

[2] Entry of project title should be by number coding (i.e., 1,2,...) and the full titles should be identified according to number at the bottom of the form (i.e., 1. full title; 2. full title....)

[3] Include NSF, other Federal and State Agencies, and private sources.

1. A Computer-Aided Design System for Castings.
2. Modeling of Transient Heat Pipe Operation.
3. Development of an Emultion-Simulation Thermal Control Model for Space Station Appl.
4. Feasibility Analysis of Reciprocating Magnetic Heat Pumps.
5. High Field, Super Conducting Magnet Laboratory Equipment.
6. Dual Energy Gamma Ray System for Bulk Density and Water Content Measurements in Porous Media.
7. Thermal Stability Limitations for Cyclically Loaded Power Transmission Systems.
8. A Computer-Aided Design System for Castings - University-Industry Cooperative Research Program.

SUMMARY OF ALL CURRENT AND PENDING RESEARCH SUPPORT [FROM WHATEVER SOURCE]

The following information should be provided for each investigator and other senior personnel (see p. 6). Failure to provide this information may delay consideration of the proposal.

	A	B	C	D	E[1]		F
	Source of Support[3]	Project Title[2]	Award Amount (or Annual Rate)	Period Covered By Award	Person-Months Or % of Effort Committed To The Project		Location Where Research Is/Will Be Performed
					ACAD.	SUMM.	
I. (Name of Principal Investigator) C. W. Meyers							
A. *Current Support* List—If none, Report none	NSF_1	Δ	$454K	'83-'86	10%	10%	ATL.
	See below for other current support						
B. *Proposals Pending*							
1. List this proposal	See A1	—	—	—	—	—	—
2. Other pending proposals, including renewal applications. If none, report none.	NSF_2	*	364K	'85-'86	20%	25%	ATL.
	NSF_3	†	163K	'85-'88	30%	65%	ATL.
3. Proposals planned to be submitted in near future. If none, report none.	—	—	—	—	—	—	—
II. (Name of co-principal investigator and/or faculty associate)	—	—	—	—	—	—	—
A. NSF_1 - Berry, Desai, Hartley, et alia							
B. NSF_2 - Berry, Desai, Hartley, et alia							
C. NSF_3 - Meyers							
III. Transfer of Support If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.	—	—	—	—	—	—	—
IV. (Other agencies to which this proposal has been/will be submitted)	—						

[1] Nonacademic researchers may report percentage of total research effort using the first column only.

[2] Entry of project title should be by number coding (i.e., 1,2,...) and the full titles should be identified according to number at the bottom of the form (i.e., 1. full title; 2. full title....)

[3] Include NSF, other Federal and State Agencies, and private sources.

Other current support:						
GT Internal	ø	$1.25K	'85	4%	0%	ATL.
Exxon	§	7.85K	'85	25%	0%	ATL.

Δ NSF_1 - A Computer-Aided Design System for Castings

* NSF_2 - A Computer-Aided Design System for Castings - University-Industry Cooperative Research Program

† NSF_3 - Evaluation of Fracture Toughness from Microstructure

ø GT Internal - Dispersed Graphite Irons

§ Exxon - Young Faculty Development Grant

A. Newcomer

SUMMARY OF ALL CURRENT AND PENDING RESEARCH SUPPORT [FROM WHATEVER SOURCE]

The following information should be provided for each investigator and other senior personnel (see p. 6). Failure to provide this information may delay consideration of the proposal.

	A	B	C	D	E[1]		F
	Source of Support[3]	Project Title[2]	Award Amount (or Annual Rate)	Period Covered By Award	Person-Months Or % of Effort Committed To The Project		Location Where Research Is/Will Be Per-formed
					ACAD.	SUMM.	
I. (Name of Principal Investigator) A. Newcomer							
A. *Current Support* List—If none, Report none	None	—	—	—	—	—	—
B. *Proposals Pending*							
1. List this proposal	NSF_1	Δ	$454K	'83-'85	20%	20%	ATL.
2. Other pending proposals, including renewal applications. If none, report none.	—	—	—	—	—	—	—
3. Proposals planned to be submitted in near future. If none, report none.	—	—	—	—	—	—	—
II. (Name of co-principal investigator and/or faculty associate)	—	—	—	—	—	—	—
A. NSF_1 - Berry, Hartley, Colwell, Desai, Meyers, Stallybrass							
B.							
III. Transfer of Support If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.	—	—	—	N/A	—	—	—
IV. (Other agencies to which this proposal has been/will be submitted)	None						

[1] Nonacademic researchers may report percentage of total research effort using the first column only.

[2] Entry of project title should be by number coding (i.e., 1,2,...) and the full titles should be identified according to number at the bottom of the form (i.e., 1. full title; 2. full title....)

[3] Include NSF, other Federal and State Agencies, and private sources.

Δ A Computer-Aided Design System for Castings

SUMMARY OF ALL CURRENT AND PENDING RESEARCH SUPPORT [FROM WHATEVER SOURCE]

The following information should be provided for each investigator and other senior personnel (see p. 6). Failure to provide this information may delay consideration of the proposal.

	A	B	C	D	E[1]		F
	Source of Support[3]	Project Title[2]	Award Amount (or Annual Rate)	Period Covered By Award	Person-Months Or % of Effort Committed To The Project		Location Where Research Is/Will Be Performed
					ACAD.	SUMM.	
I. (Name of Principal Investigator) M. P. Stallybrass							
A. *Current Support* List—If none, Report none	NSF_1	Δ	$454K	'83-'86	10%	10%	ATL.
B. *Proposals Pending*	See A						
1. List this proposal	—	—	—	—	—	—	—
2. Other pending proposals, including renewal applications. If none, report none.	—	—	—	—	—	—	—
3. Proposals planned to be submitted in near future. If none, report none.	—	—	—	—	—	—	—
II. (Name of co-principal investigator and/or faculty associate)	—	—	—	—	—	—	—
A. NSF_1 - Berry, Desai, Hartley, Newcomer, Colwell							
B.							
III. Transfer of Support If this project has previously been funded by another agency, please list and furnish information for immediately preceding funding period.	—	—	—	—	—	—	—
IV. (Other agencies to which this proposal has been/will be submitted)	—						

[1] Nonacademic researchers may report percentage of total research effort using the first column only.

[2] Entry of project title should be by number coding (i.e., 1,2,...) and the full titles should be identified according to number at the bottom of the form (i.e., 1. full title; 2. full title....)

[3] Include NSF, other Federal and State Agencies, and private sources.

Δ A Computer-Aided Design System for Castings (NSF_1)

PROGRESS TO DATE

The following brief progress report summarizes the contribution of the Georgia Institute of Technology team to the collaborative effort between the two schools. In addition to the activities in connection with the continuing meetings of the project Advisory Board, several liaison meetings between team members or the co-principal investigators have taken place, and several technical presentations to national and international bodies have been made.* In particular, the co-directors were invited to present the Official Exchange Paper for the United States at the November 1983 International Foundry Congress of the Confédération Internationale des Associations Techniques de la Fonderie in Cairo, Egypt. They also spoke at an International Workshop held at that Congress on Casting Solidification Simulation.

Funding was received for this second phase of the project during March of 1983, and commenced at the opening of the Spring Quarter in April 1983. The research has continued since that time and is essentially on schedule as originally planned.

The faculty concerned with the tasks being undertaken at Georgia Tech are detailed in Table A, along with the University of Michigan faculty involved. The individual task progress reports are appended in a later part of the continuation proposal.

---

*See listing of publications in Appendix I.

TABLE A

LIST OF TASKS AND CURRENT FACULTY TEAM MEMBERS

Task	Title and Associated Personnel	
I.	Design and Construction of a Geometric Modeler for CAD of Metal Castings	
	J. A. M. Boulet	Georgia Institute of Technology
	J. T. Berry	Georgia Institute of Technology
II.	Characterization of Transient Effects in Current and Future Molding Media	
	J. G. Hartley	Georgia Institute of Technology
III.	Special Software for Simulation of Heat Transfer in Metal Castings	
	J. O. Wilkes	University of Michigan
IV.	Transient Thermal Convection in Gating Systems	
	P. V. Desai	Georgia Institute of Technology
V.	Acoustic Emission Techniques for Monitoring Solidification	
	J. Frederick	University of Michigan
VI.	Characterization of Mold Wall Movement and Casting Thermal Gradients	
	R. A. Flinn	University of Michigan
	P. K. Trojan	University of Michigan
VII.	Control and Prescription of Heat Flux at the Casting-Mold Interface	
	G. T. Colwell	Georgia Institute of Technology
	C. W. Meyers	Georgia Institute of Technology

A. COLLABORATIVE EFFORTS

Rationale for Collaboration

The program utilizes the resources of two major universities, principally the Departments of Materials and Metallurgical Engineering and Chemical Engineering at the University of Michigan and the School of Mechanical Engineering at the Georgia Institute of Technology. Since the program is a multidisciplinary one involving the areas of transport phenomena, computer-aided design, computation techniques, solidification metallurgy and mold material characterization, the skills of a large number of individuals must necessarily by involved. The individuals within the two institutions concerned represent some of the most experienced researchers in the areas involved, working together with innovative younger contributors to the various disciplines represented.

The project co-directors have worked together over a number of years in the area concerned and are keenly aware of the complementary strengths at their respective institutions. The co-directors earlier collaborated with Advisory Board Members W. Erickson, former Chairman of the AFS Heat Transfer Committee, and Dr. C. H. Jacobs on a state-of-the-art review which was presented at the 1977 Second International Solidification Conference in Sheffield, England, by Dr. Jacobs. A further joint University of Michigan/Georgia Institute of Technology paper was delivered, this time by Professor Berry at the September 1980 Third International Solidification Conference at Warwick University, England. Most recently the co-directors presented the official U.S. Exchange Paper at the International Foundry Congress in Cairo, Egypt.

They also participated in the International Solidification Simulation Workshop held during the Congress.

The project directors have also been able to assemble the nucleus of a steering committee from their contacts with the AFS Heat Transfer Committee. The composition of this committee has been expanded to include wider industry representation.

Advisory Board and Meetings

The project Advisory Board formed includes members of the American Foundrymen's Society Heat Transfer Committee and other representatives from various segments of the foundry industry with interests in computer-aided design, as well as observers from U.S. Defense Department Agencies having computer-aided design of casting interests. A representative of the National Science Foundation Division of Applied Research has frequently been present at Advisory Board meetings.

A list of Advisory Board members and of the guests at the October 1983 meeting is attached.

Advisory Board
for
NSF Funded Project:
A Computer-Aided Design System for Castings
underway at
The Georgia Institute of Technology
and
The University of Michigan

A. Umble*	Bethlehem Steel Corporation
W. Erickson	Los Alamos Laboratories
R. W. Ruddle*	Consultant
O. Riegger*	Tecumseh Products
M. Robinson*	John Deere and Company
G. Cole	Ford Motor Company
T. Watmough	International Harvester Company
S. Davis	Esco, Incorporated
R. Warrick	Lynchburg Foundry
R. Martin	FMC Corporation
C. H. Jacobs	Zimmer, Inc.
D. Stein	Anacast Division, Anadyne Corporation
L. Wilson	Higgins Foundry
G. K. Ruhlandt	General Motors, Manufacturing Research
S. D. Sanders	Caterpillar Tractor Company
C. Zanis	D. Taylor Ship R & D Center, U.S. Navy

Guests+:

R. Skrocki	TRW, Inc.
R. Spear	Alcoa Laboratories
M. Walter	Abex Corporation
W. Andresen	American Die Casting Institute
F. Bliss	Ford Motor Company
F. Jackson	Zimmer, Inc.

*Members of AFS Heat Transfer Committee

+At October 1983 Advisory Board Meeting.

B. RESEARCH PROGRESS

Progress Report on Task I: Design and Construction of a Geometric Modeler for Computer-Aided Design of Metal Castings

Summary of Progress:

The goal of Task I is to provide geometric models that can conveniently be linked to and used by the special software being developed in Task III for solidification simulation. Each geometric model will exist as a computer database, and Task I will provide software for convenient retrieval of information from these databases. The primary use of this information will be in generating finite element meshes for use in the Task III simulations. Mesh generation will be accomplished jointly by the Task I and Task III team members. Details of the procedure will not be clear until the simulation software is further along. If appropriate graphical display devices are available, interactive software to help with the mesh generation will be developed.

Several recent accomplishments have prepared us for the work described above.

1. A commercially available geometric modeler (CAT-1) has been interfaced with a two-dimensional (2-D) finite element heat conduction simulation program [Dalton, 1983]. The resulting software system allows interactive model generation, part sectioning, fully automatic mesh generation, interactive specification of boundary conditions, simulation of transient heat conduction, and automatic drawing of isotherms. The software operates on a TERAK minicomputer system with color monitor.

2. Several methods of accounting for latent heat release during solidification have been implemented in a one-dimensional, finite element solidification simulation program [Raggi, 1984]. One of the algorithms implemented is that proposed by Wilkes [1983] for use in Task III. This same algorithm is being implemented in the 2-D simulation program referred to above.

3. A commercial (Lynchburg Foundry) casting, with rigging, has been modeled using TIPS-1. This work has been originally discussed by Berry and Boulet [1983].

4. The latest version of the TIPS modeler (TIPS-2) has been acquired from Cornell University.

5. Equipment grant requests for a state-of-the-art three-dimensional (3-D) graphical display have been submitted to the Georgia Tech Research Institute and the National Science Foundation.

6. The College of Engineering has been granted $\sim$\$2M in computer-aided design (CAD) equipment from IBM. This equipment includes an IBM 4341 computer and several graphical display devices. The unit is currently being installed and should be on line by May 1, 1984.

7. The School of Mechanical Engineering has acquired a VAX 11/750 computer dedicated to support of robotics and CAD. The unit was recently commissioned.

Summary of Proposed Work:

Several specific steps toward achieving the goals discussed above are planned for the coming year. These include: (a) implementation of TIPS-2 on the IBM 4341, (b) acquisition of PADL-2 and implementation on the VAX 11/750, (c) evaluation and comparison of TIPS-2 and PADL-2 database structures as regards their suitability for the purposes of Task I, (d) design and implementation of the data retrieval software required for Task I, (e) close monitoring of the software being developed in Task III. The data retrieval software may be either a stand-alone program or an addition to one of the modeling programs.

By the end of the coming year, ground work will have been laid for development of a mesh generation interface between one of the modelers and the Task III software. This development is scheduled for the third year of the project.

References

J. T. Berry and J. A. M. Boulet (1983), "The Application of Geometric Modeling to Metal Casting Technology," Proc. of G. M. Symposium of Solid Modeling by Computers.

B. Dalton (1983), M. S. Thesis, Georgia Institute of Technology, School of Mechanical Engineering.

E. Raggi (1984), M. S. Thesis, Georgia Institute of Technology, School of Mechanical Engineering.

J. Wilkes (1983), Private communication.

Progress Report on Task II: Characterization of Transient Effects in Current and Future Molding Media

Summary of Progress

The two areas of primary concern in Task II are the development of a predictive model for the thermal conductivity of bonded molding sands and the analysis of the heat and moisture transport in green sand molds. The predictive model is considered essential because determination of the influence of sand type, initial moisture content, binder content, ramming density and temperature on the effective thermal conductivity of bonded sands by experimental methods alone is essentially an insurmountable task. The analysis of the transport mechanisms in green sands is important because the current state of knowledge is insufficient to predict or describe the increased chilling power observed when moisture is present in mold materials.

Initial efforts related to model development for thermal conductivity were predicated on the assumption that the unit-cell model developed by Jackson [19] could be modified for use with clay binders. However, a careful analysis of the model as well as comparison of model predictions and experimental measurements of the thermal conductivity of fluid-saturated sands (two-component systems) revealed that the unit-cell model does not adequately account for the effect of the volume fraction of sand. Furthermore, similar comparisons for a number of models available in the literature has shown that none of the models were entirely adequate for predicting the thermal conductivity of two-component systems.

Therefore, a new two-component model, called the cylinder-element model, has been developed under this grant to predict the thermal conductivity of fluid-saturated sands. As the geometrical characteristics of sands can be described statistically but not analytically, the influence of particle size, size distribution and shape are determined empirically for use with the cylinder-element model.

The first phase of the experimental program used to complete and to verify the cylinder-element model has been completed. This effort included thermal conductivity measurements on a silica sand with round particles (Ottawa silica) and an angular silica sand (Masonry sand) at various densities and with several saturating fluids having vastly different thermal conductivities (air, water, transformer oil and ethylene glycol). These measurements were used to establish two geometrical parameters which describe the influence of particle size and shape.

The results obtained from the model development phase of Task II can be summarized as follows:

1. The cylinder-element model is capable of predicting the thermal conductivity of two-component systems within about two percent of values determined experimentally.

2. The requisite geometrical parameters of a sand for use in the cylinder-element model for two-component systems can be determined from three experimental thermal conductivity measurements.

3. The cylinder-element model can also be used to evaluate the thermal conductivity of solid (sand) particles when this value is unknown.

4. The cylinder-element model can be extended for use with three-component systems, such as bentonite-bonded sands (bentonite-sand-air).

The analysis of heat transport in green sands has also been initiated during this phase of work on Task II. An analytical model, based upon the conservation of mass, energy and momentum, has been developed. This model would be used to predict the temperature, moisture and pressure distribution in a green-sand mold during the solidification of castings.

Summary of Proposed Work

During the next phase of Task II, the development of the predictive model for thermal conductivity will continue. The cylinder-element model, valid for two-component systems, will be extended to predict the behavior of three-component systems. In particular, a three-component system whose binder is bentonite clay will be examined in detail. Considerable experimental work is necessary to achieve this goal. In addition, a method for measuring the thermal conductivity of the pure bentonites must be developed. Associated tasks include the refinement of the high-temperature probe used to measure the apparent thermal conductivity of sands at elevated temperatures and expanding the capability of the cylinder-element model to account for the influence of temperature on the apparent thermal conductivity of bonded sands.

The analysis of heat and mass transport in green sands will also continue. Implementation of the recently developed model in a finite difference computer program is planned. In addition, experimental work, including an instrumental test section compacted with green sand is planned for construction. Experimental measurements of temperature and drying in green sands will be used to validate and refine the analytical model.

References

K. W. Jackson (1980), Ph.D. Thesis, Georgia Institute of Technology, School of Mechanical Engineering.

Progress Report on Task IV: Transient Thermal Convection in Gating Systems

Summary of Progress

The research under this task centered on developing a predictive computational model to calculate transient forced convection heat transfer during the pouring-filling stage of a casting. Figure 1 shows a schematic of the simplified transient heat transfer model. The superheated metal entering the channel at a constant temperature progressively loses its heat to the mold and the surroundings as it flows. The model for such a process must include:

a. the classical thermal contact problem,

b. the moving boundary problem, and

c. the moving free surface problem.

This heat transfer process is a two-phase (solid skin-liquid melt), two-media (mold-metal), moving thermal contact (flowing metal contacting the mold), moving free surface (of the liquid metal) problem that is further complicated by the geometry of the runners and the risers and the ingates and so forth. Therefore, it was decided to examine individual aspects of the problem, one at a time, without losing sight of the needs of the modern foundry practice to apply the results of the overall problem solution in the not-too-distant future.

The moving boundary, free surface heat transfer problem was examined by first considering the transient thermal contact problem between the hot metal filling the system instantly and the colder mold walls. This was followed by examining the overall transient forced convection process including the advancing free surface.

The moving boundary thermal contact problem was solved by developing a novel finite element computation methodology. In particular, a fictitious layer of finite elements was introduced at the interface between the mold and the liquid metal flowing in a runner channel. The layer was assigned fictitious thermal properties of very high thermal conductivity and a very low thermal capacity. The actual values of the thermal properties were chosen by numerical experimentation that also included a variation of the grid size, Δx, and computational time step, Δt, to render an optimum Fourier number, $\alpha\Delta t/(\Delta x)^2$, of the fictitious layer, α being the thermal diffusivity of the liquid metal.

The influence of forced convection due to the metal velocity was incorporated into the fictitious layer model of the instantly filled channel. For sample calculations, the liquid velocity was taken as 100 cm/sec in a channel of 20-cm length and 4-cm width. The thermal properties for the liquid and the mold refer to those of liquid aluminum and mold sand, respectively. Typical numerical results are shown in Figure 2. In this figure, T is the variable fluid temperature evaluated at the interface contact and T_{01} and T_{02} are the initial temperatures of the metal and the mold, respectively. Numbers 1, 2, 3, and 4 denote locations on the interface at respective distances of 5, 10, 15 and 20 cm from the entrance. In other words, the fluid which was at the channel entrance when the heat transfer began extends up to point 1 at time t = 0.05 second; at t = 0.1 second this fluid reaches point 2 and so forth. It may be noted that until the fluid which was at the channel entrance (when the heat transfer process started) reaches a given exial position, the interface temperature beyond that position remains approximately equal to the analytically predicted value in the one dimensional case. After 0.2 second, all locations inside the channel are affected by convection, which causes the interface temperature to rise. In the context of industrial thermal processing systems, such as metal castings, such an initial rise of the interface temperature is a noteworthy phenomenon. Further details of this work are available in a paper by Kim and Desai, details of which are given in Appendix I.

Calculations of heat transfer to the initially cold empty mold channel from the advancing hot liquid metal required a simultaneous calculation of the conjugated temperature fields within the mold and the metal. The model to calculate the mold/metal interface temperature was developed by first writing the conservation statements for a control volume whose surface moved with an arbitrarily specified velocity field on it and then developing the corresponding finite element model. This formulation included the effect of control volume deformation. The conventional Galerkin procedure becomes applicable to the deforming control volume finite element analysis, without any need to assume the deformation to be temporarily stationary during each computational time step.

The interface temperature distribution due to forced convection to the instantly filled channel, discussed earlier and shown in Figure 2, is a special case of the general solution when mesh deformation vanishes. A comparison of results obtained by the fixed and the variable mesh formulations for the instantly filled case is shown in Figure 3. The close agreement seen in the results establishes a rationale to adapt the deforming control volume analysis for the moving free surface heat transfer problem.

Summary of Proposed Work

An excellent agreement between the results obtained by employing a deforming mesh model with those obtained via a fixed mesh for the instantly filled channel provides a sound basis to adopt the continuously deforming finite element scheme for further work on moving free surface heat transfer. It is expedient to employ this model to generate further information of significance to the practicing foundry engineer. For example, a parametric study needs to be undertaken to assess the influence of mold and liquid metal material properties, the filling rate of the runner system and the channel geometry on the heat loss characteristics of the runner channel.

Secondly, the presently available model is only applicable to straight horizontal channels. The deforming mesh generation scheme must be reexamined so as to include a gradual variation in channel cross-section. In addition, the influence of gravity must be explicitly included for such non-horizontal channels as the downsprue. This will lead to quantitative evaluation criteria for the thermal design of sprues. In other words, the influence of taper, length and aspect ratio of the sprue, in addition to the material (mold and liquid metal) properties on the thermal behavior of sprues can then be quantitatively assessed. The influence of gradual change in the flow direction within the gating system must also be examined in such an analysis.

There are other aspects of the gating system design, such as solidification during filling and sudden changes in cross section, which are of critical concern from the applications point of view. However, as explained earlier, the total problem of convection currents during the entire casting cycle is quite complex and must be handled in gradual steps for detailed consideration of each aspect. Nevertheless, efforts will be made to propose changes in the computational model which can clarify the issues involved.

Briefly, the proposed research will undertake:

(a) a parametric study of the deforming finite element model already developed;

(b) a modification of the mesh generation scheme to include gradual variation in channel cross-section;

(c) to include the effect of gravity and flow direction changes within the runner system; and

(d) to assess the thermal design of tapered sprues.

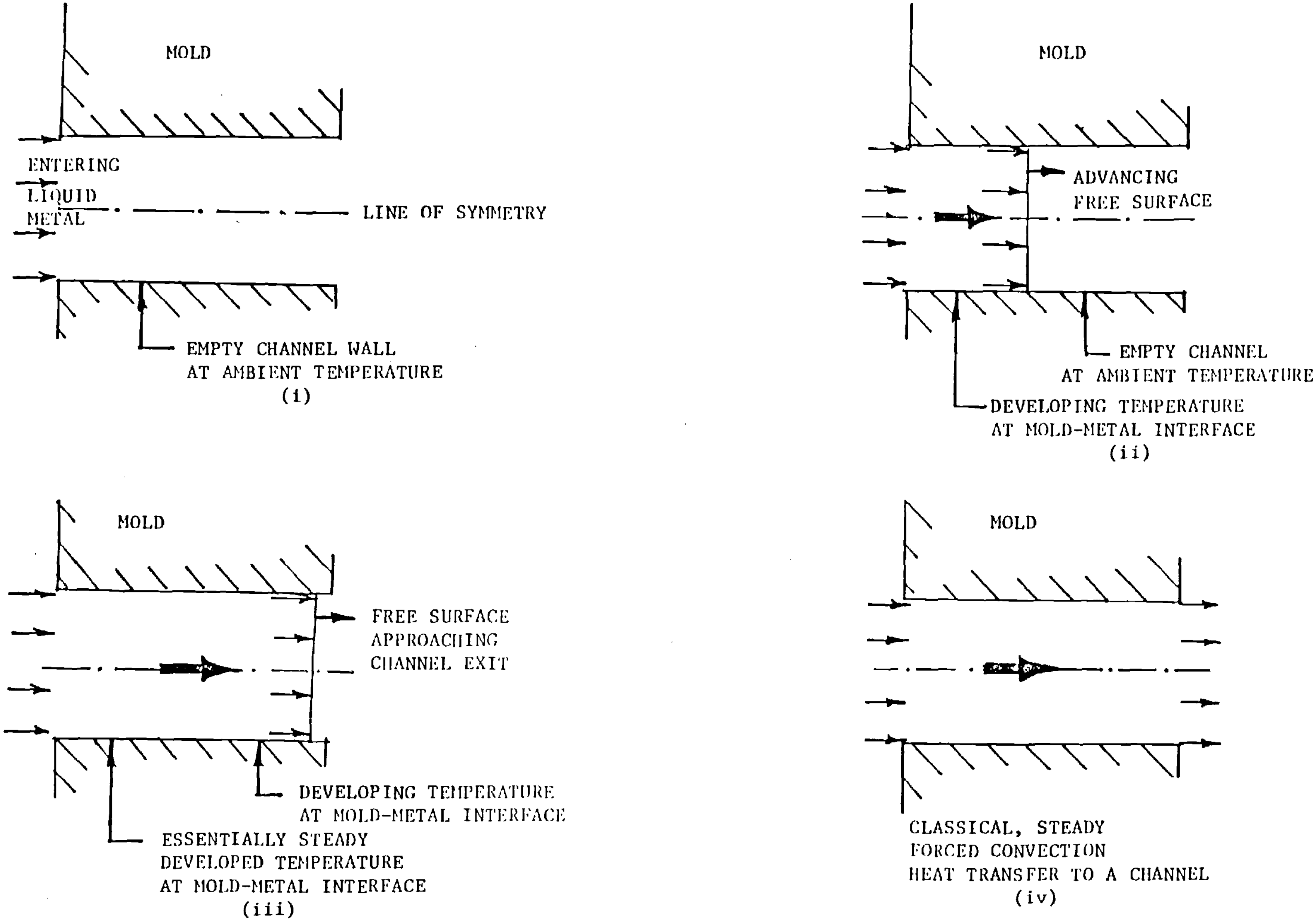


FIGURE IV-1. Schematic of Transient Heat Transfer Model.

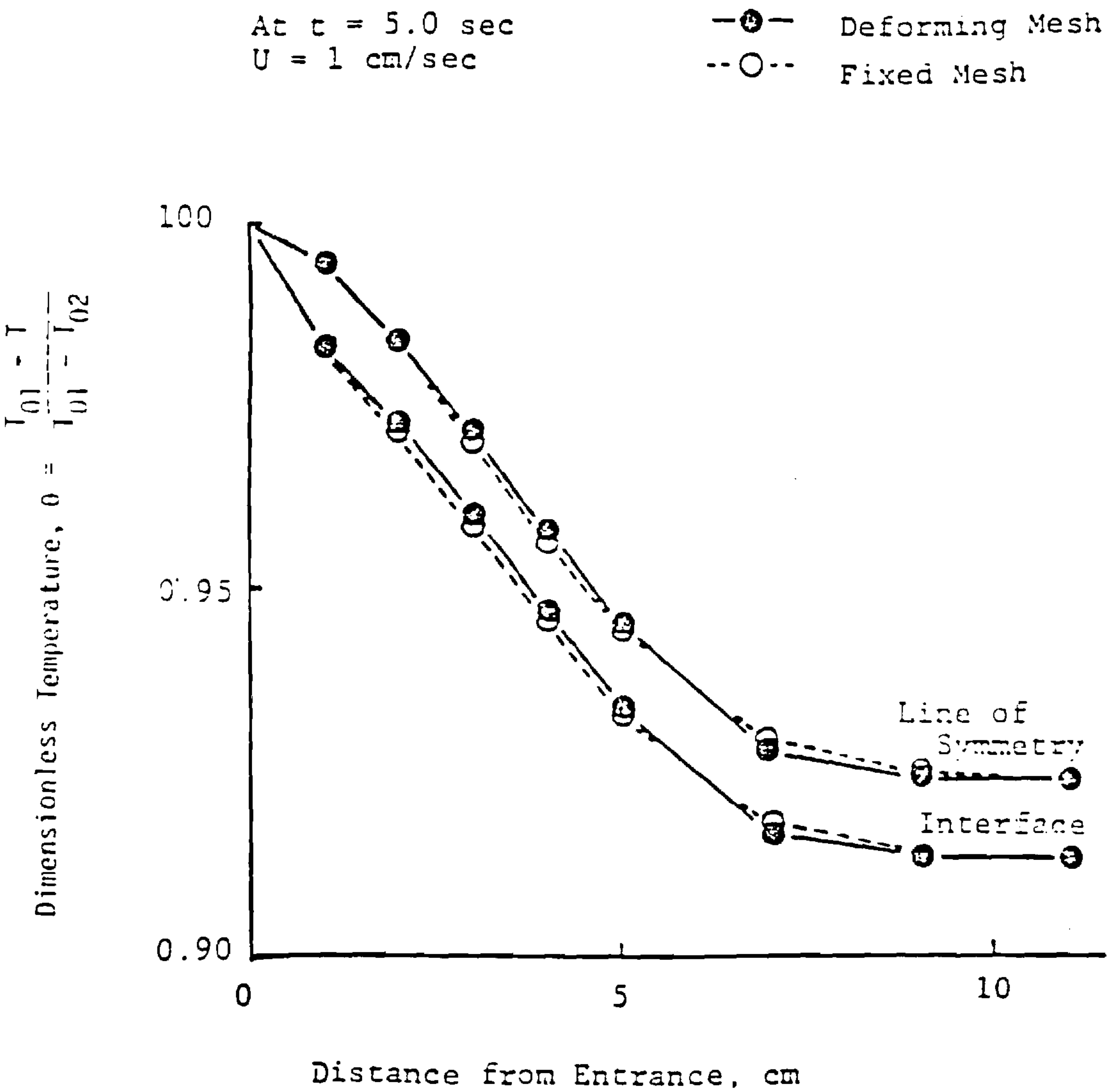


FIGURE IV-2. Influence of Forced Convection on Mold-Metal Interface Temperature for an Instantly Filled Channel.

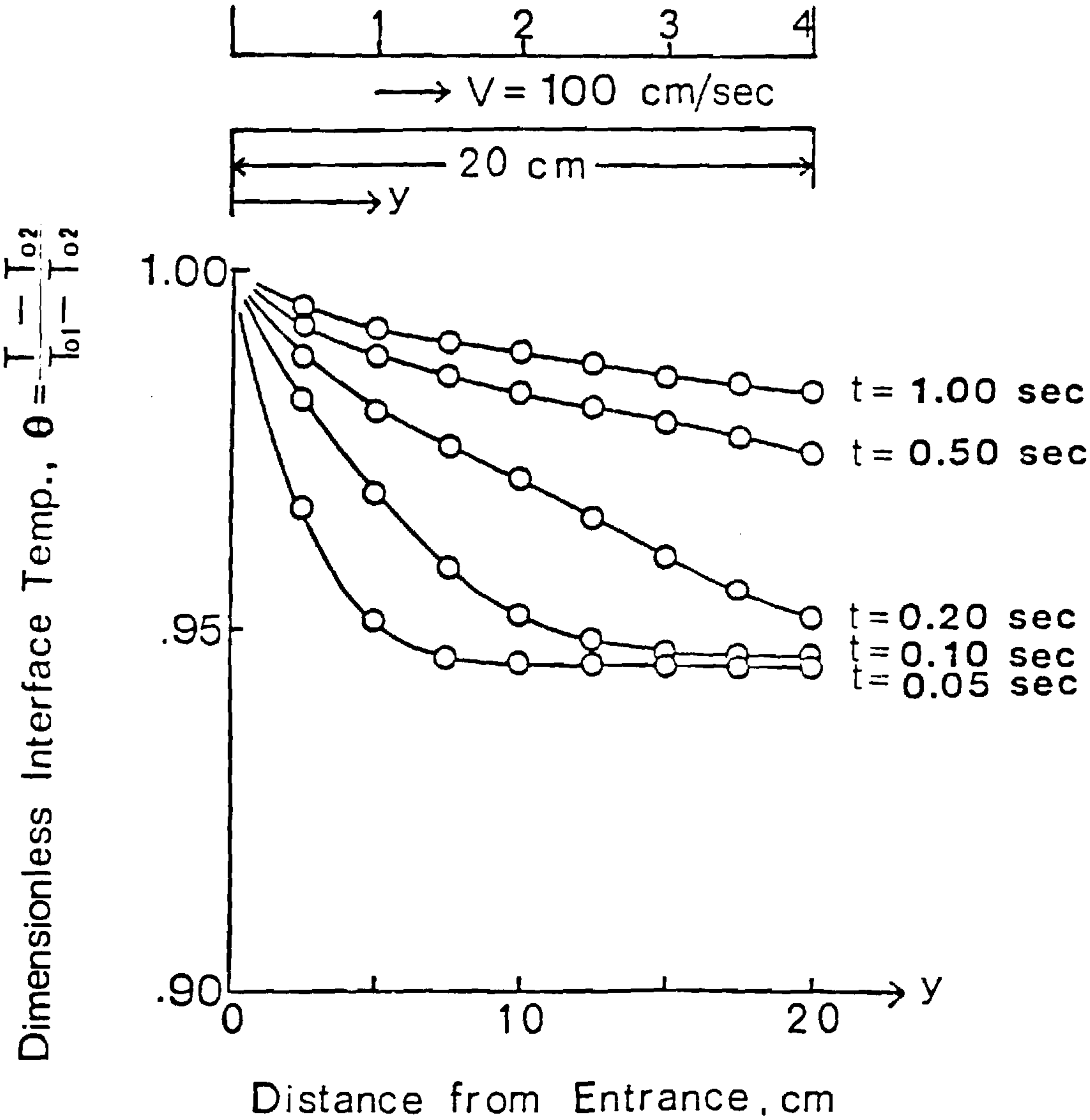


FIGURE IV-3. Comparison of Temperature Computations via Fixed and Deforming Mesh Formulations.

Progress Report on Task VII: The Control and Prescription of Heat Flux at the Casting-Mold Interface

Summary of Progress:

Heat pipes have a remarkable ability to transport thermal energy through relatively great distances with very small temperature drop. They may be designed to operate anywhere between very high (2000°K) to very low (nearly 0°K) temperatures and they can be made in a variety of shapes. The conductance of a given heat pipe can be controlled by introducing or removing non-condensable gas to the working fluid region.

In view of these characteristics it would seem logical that heat pipes might be used to good advantage to control casting solidification profiles, cooling rates and hence enhance the quality of cast products.

The present program of work started under former Task VIII of the previous grant and has continued over the first year of the present funding. The initial study, summarized in the third progress report of the previous grant, was concerned with the feasibility of applying a typical heat pipe assembly to a two-dimensional rectangular insulated mold enclosure. Attached to the mold enclosure was a heat pipe (side face) and a conventional water cooled chill (lower face). The model constructed described not only the shape and orientation of the freezing front but the effects of the operating variables of the heat pipe. It was shown that the presence of the heat pipe could strongly affect the shape of the freezing front. Furthermore, since the heat pipe could be activated at any desired instant, it was seen that the control of the heat flux-time relationship was a most promising aspect of the application of this device.

One of the limitations of the preliminary model was the treatment of the interfacial regions occurring between the heat pipe and the molten metal alloy, and also between the chill and the alloy, where good thermal contact was assumed.

During the first year of the present grant period, the model was expanded to include a temperature dependent thermal resistance as might be expected in cases where an air-gap forms after solidification at the mold-metal interface. This was undertaken for both the chill and the heat pipe metal interfaces. The formulation concerned permitted the inclusion of gases other than air into the gap concerned. Consideration was also given to the time of formation and to the size of the air-gap concerned. Comparisons with practical data provided by Durham (1979) for water cooled chills were used to validate the gap modeling. Good agreement between predicted data and experimental information can be obtained if both temperature and gap thickness is accounted for. A Master's thesis (A. Lodhia, 1984) was recently read, which discusses this work more fully.

<u>Summary of Proposed Work</u>

The original terms of reference of the second phase of this task embrace not only the use of heat pipes in controlling and prescribing heat-flux at the mold-metal interface but the study of the heat-flux profiles that occur in complex geometrical features of shaped sand castings. In as much as these features frequently contribute to the sequential aspects of solidification, through end and corner effects, it is important that the effects of such features be understood. A pioneering effort in this direction has been the work initiated under former Task IIb (the Q-method approach). A series of papers* have described how a boundary heat-flux related function can be used to replace the many nodes required to simulate the body of the mold in finite difference or finite element computations of freezing front progress. Work will continue in this area under the aegis of Task VII. In particular, the degree of interaction of geometrical features with each other and the relationship of two-dimensional to three-dimensional features will be explored. One particular aspect that will be examined will be riser or feeder-head cavities, where re-entrant corners are sometimes utilized to postpone solidification, as an alternative to insulating sleeves or exothermic compounds. Attempts will be made to demonstrate how such adjuncts operate and how their design can be optimized and thereby casting yield increased. The work will involve application of the Q-method during simulation, together with an experimental validation involving several aluminum castings, which will be poured in dry-sand molds.

Heat pipe related work will also continue along the lines originally envisaged. Having now modeled air-gap effects, an effort will be made to design and build a mold assembly containing a heat-pipe device in order to further validate the total model system. Initial runs here will be undertaken with the Pb-Sn alloy simulated in the model experiments. Both heat-pipe parameters and the superheat of the molten alloy will be varied during these activities.

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*See Appendix I.

References

D. Durham (1979), "Numerical Simulation of Solder Solidification," Welding Research Supplement, Welding Jnl., October, pp. 301-305.

A. Lodhia (1984), M. S. Thesis, Georgia Institute of Technology, School of Mechanical Engineering.

PROGRAM OBJECTIVES FOR YEAR II

The research objectives in the four major task or program areas are outlined in the following paragraphs.

The goals for Task I are primarily those of determining the potential of TIPS-2 and PADL-2 as bases for a castings technology dedicated geometric modeler, which can be interfaced with the special purpose computational software being developed at the University of Michigan under Task III. At an appropriate point, a comparison will be made of the TIPS-2 and PADL-2 data base structures as regards their suitability for the purposes envisaged. Close liaison will be maintained with the Task III team during all phases of the work program.

The goals of Task II will embrace both experimental and theoretical aspects. Thermal conductivity determination of mold materials and their components at elevated temperature will continue. In particular the thermal conductivity-temperature data will be compiled for pure bentonites. This will then enable work to proceed further with the development of the cylinder-element predictive model for the thermal conductivity of bentonite-bonded dry sands.

Work will also continue on the analysis of heat and mass transport in green sands, utilizing a recently developed formulation for which a finite difference computer program is to be developed. Experimental work in connection with the validation of this model is also planned.

The goals for Task IV for the coming year will center initially upon the expansion and application of the continuously deforming finite element scheme evolved to study the effects of mold and metal media properties, filling rates and channel geometry on heat loss characteristics of the runner channel.

The scheme will then be applied to the case of the vertical tapered downsprue in a similar fashion. Other more complex aspects, for example sudden cross-section changes, will be studied to determine whether the present computational model is applicable.

Finally, Task VII will now be oriented to furthering the understanding of heat flux characterization for the boundaries of more complex casting shapes than those analyzed heretofore under the former Task IIb. In particular the interaction between neighboring corner details will be examined.

Regarding the eventual use of heat-pipe related devices in the control of boundary heat flux during solidification, work will be continued in this area, with a mold-assembly containing a heat-pipe cooled zone to be designed, constructed and tested. It is to be hoped that the assembly will prove useful in validating further the computer model now available for describing a heat-pipe cooled "window" in a mold cavity.

E-25-602

PROGRESS REPORT NO. 1

A COMPUTER-AIDED DESIGN SYSTEM FOR CASTINGS

Prepared by:

CADCAST PROJECT TEAM
School of Mechanical Engineering
Georgia Institute of Technology

Prepared for:

NATIONAL SCIENCE FOUNDATION
Division of Mechanical Engineering and Applied Mechanics
Attention: Dr. William M. Spurgeon

Under

Grant No. MEA 82 11524

November 1984

GEORGIA INSTITUTE OF TECHNOLOGY
A UNIT OF THE UNIVERSITY SYSTEM OF GEORGIA
SCHOOL OF MECHANICAL ENGINEERING
ATLANTA, GEORGIA 30332

A COMPUTER-AIDED DESIGN SYSTEM FOR CASTINGS

Progress Report No. 1

Prepared by:

Project Team Tasks I, II, IV and VII
School of Mechanical Engineering
Georgia Institute of Technology

Covering the Period from
February 1983 through April 1984

Prepared for:

National Science Foundation
Mechanical Engineering and Applied Mechanics Division

Attention: Dr. William M. Spurgeon
Grant No. MEA 82 11524

October 1984

CONTENTS

CONTENTS (Continued)

CONTENTS (Continued)

ABSTRACT

A large scale investigation of the problems associated with the computer-aided design of castings has been conducted by collaboration at two major academic institutions. This report comprehensively summarizes the work completed during the first year of a second three-year funding period at the Georgia Institute of Technology and the University of Michigan.

Seven task areas have been selected for investigation which relate to the major scientific or engineering related roadblocks in this area. These roadblocks pertain to the geometric modeling/physical simulation problems, the provision of thermal transport data, the control and prescription of interfacial heat flux during solidification, the problem of filling transients associated with pouring of castings, the modeling of interfacial phenomena, the accurate description of the interaction of the molding medium and the solidifying casting and finally the total economics of the computation system itself.

The resources of the two academic institutions have been brought to bear on these tasks over a six year funding period. The investigation has been guided by a blue-ribbon steering committee representing all aspects of the industry, its technology-related trade associations and several government agencies. The investigation should prove to have a major impact upon both the productivity and product quality of the nation's metal casting industry.

ACKNOWLEDGEMENTS

The current program is being sponsored by the Mechanical Engineering and Applied Mechanics Division of the National Science Foundation under Grant Number MEA 8211084. During the past three years William M. Spurgeon has served as Project Manager. His valuable guidance in the course of the investigation is gratefully acknowledged.

The present report, which reviews progress over the first year of a second three-year program, is the result of the effort of several individuals associated with the various tasks, as well as the overall aspects of the project:

Professor J. T. Berry, Co-Principal Investigator

Professor P. V. Desai, Co-Investigator, Task IV

Professor J. A. M. Boulet, Co-Investigator, Task I

Professor J. G. Hartley, Co-Investigator, Task II

Professor C. W. Meyers, Co-Investigator, Task IV

Professor G. T. Colwell, Co-Investigator, Task VII

The following students participated in the experimental and computational efforts:

J. Moosbrugger, Task VII

V. Gourisankar, Task I

C. Kim, Task IV

S. Park, Task II

P. Krishnan, Task IV

The manuscript was prepared by Mrs. Terri Parise who held the overall responsibility of assembling and collating the contents of the report and arranging for its distribution.

RESEARCH OBJECTIVES

A Computer Aided Design System for Castings

There are a variety of economic and technological advantages associated with the application of computer aided design to castings and casting production. The challenges involved in designing castings, including the issue of basic castability, the design of casting rigging, including provision of risers, gates, chills, and patterns, and the delivery of such designs successfully into a production system are problems which are faced in all areas of the foundry industry, including permanent mold and die casting plants.

At the outset of the current research program, a number of important scientific or engineering related "roadblocks" were recognized as being limitations to the widespread application of computer aided techniques to the solution of these problems. The principal roadblocks were identified as follows:

1. Utilizing geometrical modeling techniques and linking them successfully with simulation technologies.

2. Providing for those computations, the various thermal data required for successful simulation of both the casting and the mold behavior.

3. Evaluating associated costs of computation of the various alternative modes of simulation currently available, and providing easy access for the casting producer and designer into computer codes associated with these technologies.

4. Assessment of mold filling transients, including fluid flow and heat transfer and their interrelationships during the period of pouring and immediately thereafter.

5. Describing the various applicable boundary or interface conditions in the processes concerned and determining their order of importance.

6. Modeling those phenomena describing, for example, the interaction of thermal expansion-contraction of both casting and modeling media during freezing and feeding limitations so that distribution and magnitude of unsoundness might be predicted.

In the current program these areas have been extensively investigated by the research teams at Georgia Tech and The University of Michigan with considerable interaction between the individuals involved in each of the areas of research investigation. Considerable progress has been achieved in each of these areas and in the interfaces lying between them. These achievements are described in detail later in this report. Based on these advances, the uncovering and identification of areas where further effort can bring about a substantial forward movement in implementing computer aided design in the foundry industry has led to the continuation of research in this area. The objectives of the program involve not only an extension of several of the areas currently under investigation, but other technologies which are critical and important in the implementation and utilization of a computer based design system. Some new technologies are under investigation which involve instrumentation techniques and new solidification control technologies to further the installation of CAD in the foundry industry.

Several research objectives are being emphasized in the present study because their achievement can markedly advance CAD in this area. New research tasks to approach these challenges are incorporated in this research activity. This project is directed toward new areas and new levels of achievement necessary to accomplish the goal of implementing computer aided design of casting in the foundry industry. While some are based on current work, others are directed toward new areas to bring about the extension of computer applications in the cast metals industry.

Specifically, the objectives of the continuing research are:

1. Development of software for interfacing geometric modeling and heat transfer programs in two and three dimensions.

2. Characterizing thermal properties of current and future molding media.

3. Development of software for simulation of heat transfer in solidification of metal castings which is compatible with mini-computer systems and utilizes a simple and directly accessible input and output organization.

4. Assessing transient heat transfer in pouring, gating and mold filling.

5. Investigating acoustic emission techniques for monitoring solidification.

6. Characterizing metal and mold wall movements and the role of thermal gradients in casting solidification.

7. Studying techniques for direct control of heat flux during casting solidification.

This program is being carried out by researchers at The Georgia Institute of Technology and The University of Michigan with a high degree of cooperation and research task interaction. The research areas 1, 2, 4 and 7 are centered at Georgia Tech, and areas 3, 5 and 6 at Michigan.

This project will continue a major program to study the scientific, engineering, and economic aspects of computer aided design in the casting industry. The implementation of CAD techniques in casting development and manufacturing production is the primary objective of this program.

PROGRESS REPORT

This report summarizes progress on The Georgia Institute of Technology portion of the collaborative effort with The University of Michigan. The period covered by this report extends to April 1984. The project has been divided into seven tasks, four of which are being conducted at The Georgia Institute of Technology and are summarized in the following paragraphs (Tasks I, II, IV and VII).

The official starting date for the program was February 1, 1983.

This progress report first describes collaborative efforts between the two participating institutions. Following are reports on each of the four areas of activity at The Georgia Institute of Technology.

COLLABORATIVE EFFORTS

Rationale for Collaboration

The program utilizes the resources of two major universities, principally the Departments of Materials and Metallurgical Engineering and Chemical Engineering at The University of Michigan and the School of Mechanical Engineering at The Georgia Institute of Technology. Since the program is a multi-disciplinary one involving the areas of transport phenomena, computer aided design, computation techniques, solidification metallurgy and mold material characterization, the skills of a large number of individuals must necessarily be involved. The individuals within the two institutions concerned represent some of the most experienced researchers in the areas involved, working together with innovative younger contributors to the various disciplines represented.

The project co-directors have worked together over a number of years in the area concerned and are keenly aware of the complementary strengths at their respective institutions. The co-directors collaborated in presenting the Official USA Exchange Paper at the International Foundry Congress in Cairo, November 6-11, 1983, "Computer Aided Design for Castings".

The project directors have also been able to assemble the nucleus of a steering committee from their contacts with the AFS Heat Transfer Committee. The composition of this committee has been expanded to include wider industry representation.

Advisory Board and November 1984 Board Meeting

An Advisory Board has been formed and includes members of the American Foundrymen's Society Heat Transfer Committee and other representatives from various segments of the foundry industry with interests in computer aided design, as well as the U.S. Army and U.S. Air Force who have computer aided design of casting programs. A representative of the National Science Foundation also participates as a member of the Advisory Board.

A list of Advisory Board members and guests who have been invited to the November 1-2, 1984, meeting which is to be held in Atlanta is appended, together with an agenda.

ADVISORY BOARD

*For Gerald K. Ruhlandt

OTHER INVITEES - NSF ADVISORY BOARD MEETING

NOVEMBER 1-2, 1984
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA

William Andresen
Viking Diecast Corp.

Steve Antolovich
Georgia Tech

Mike Beffel
Univ. of Michigan

John T. Berry
Georgia Tech

Wayne Book
Georgia Tech

J. A. M. Boulet
Georgia Tech

Leo Buchakjian
General Electric Co.

Gene T. Colwell
Georgia Tech

A. D. Cotney
West Point Foundry

Wallace Day
Columbus Foundries

P. V. Desai
Georgia Tech

Randy Erickson
Dynarad Corp.

Peter Findlay
ADCI

Dr. Harold Gegel
Wright-Patterson AFB

V. Gourisankar
Georgia Tech

Dan Gulden
E. V. Camp

James Hartley
Georgia Tech

Jeff Heinen
General Electric Co.

Harvey Henderson
Consultant

Tim Hitchcock
Hitchcock Foundries

Thomas Hurley
Bodine Aluminum Co.

A. Jeyarajan
TRW

Avery Kearney
Trialco

P. Krishnan
Georgia Tech

Kimio Kubo
Osaka Univ., Japan

Arno Louvo
VTT, Finland

Charles Lowery
Lockheed

William McCollum
ITT Grinnell Corp.

Carolyn W. Meyers
Georgia Tech

John Moosbrugger
Georgia Tech

Larry Nolan
McAUTO

S. Park
Georgia Tech

Robert D. Pehlke
Univ. of Michigan

Glenn Reinemann
Morris Bean and Co.

Abe Scheinker
Northrop Corporation

Rod Scollard
Boeing

Rudy Sillen
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Roger Stafford
SDRC

Mike Stallybrass
Georgia Tech

E. E. Underwood
Georgia Tech

H. P. Wang
General Electric Co.

John White
Georgia Tech

John R. Williamson
Air Force Flight Dynamics Lab

Ben Winter
Univ. of Michigan

CADCAST 1984

ANNUAL ADVISORY BOARD MEETING
Joint GT-UM Program in Computer Aided Design
for Casting and Solidification Technology
(Supported by NSF)
Atlanta, Georgia

Thursday, November 1

8:30	Registration: SST Building Lobby	
9:00	Welcome	Dr. John Brighton*
	RESEARCH PROGRESS	
9:15	Introductory Remarks	Prof. R. D. Pehlke+
9:45	Geometric Description	Dr. Toby Boulet*
10:15	Break	
10:30	Computer Code Development	Mr. Mike Beffel+
11:00	Data Base Development	
	(i) Mold Transport Model	Dr. James Hartley*
12:00	Luncheon: Brittain T-Room	
1:30	Data Base Development	
	(ii) Expansion-Contraction Phenomena	Mr. Ben Winter+
2:00	Forced Convection Phenomena	Dr. P. V. Desai*
2:30	Experimental Techniques for Controlling and Monitoring Solidification	
	(i) Acoustic Emission Devices	Mr. Ben Winter+
	(ii) Heat Pipe Technology	Dr. G. Colwell*
3:15	Break	
3:30	Advisory Board: Communications and Discussion of Progress	
5:00	AFS Heat Transfer Committee Meeting	
7:00	Cash Bar at Sheraton-Atlanta Hotel	
8:00	Dinner meeting at Sheraton-Atlanta Hotel 590 West Peachtree Street, N.W.	

*Georgia Tech

+University of Michigan

Friday, November 2

RELATED RESEARCH

8:30	Cooperation with Overseas Groups	Prof. John Berry*
9:00	HUBERT Project of the Nordic countries	Arno Louvo, VTT, Helsinki, Finland
9:30	Japanese Developments	Dr. Kimio Kubo Osaka University, Japan
10:00	Discussion	
10:15	Break	
10:30	NOVACAST Program - A first level software package	Rudy Sillen, Novacast AB, Ronneby, Sweden
11:15	Demonstrations/Displays TIPS/PADL - NSF Program NOVACAST - Novacast AB CATSOFT - Catronix Corporation	
12:00	Luncheon: Student Center, Room 301	
1:15	Visits A. Materials Handling Research Center B. Flexible Automation Laboratory C. Heat Pipe/Heat Transport Laboratories	
2:00	Reconvene: Summary of current industry needs	Prof. Pehlke
3:00	Closure	Prof. Berry

PAPERS PRESENTED

Papers presented or published in connection with the Georgia Institute of Technology's tasks are as follows:

1. Progress Report on the Computer-Aided Design Systems Project. J. T. Berry and R. D. Pehlke, Trans. AFS (88), 1980, pp. 615-622.

2. Thermal Properties of Mold Materials. J. G. Hartley and D. Babcock, in Modeling of Casting and Welding Processes, Ed. by H. Brody and D. Apelian, AIME 1981, pp. 83-92.

3. Convection in Mold Cavities. P. V. Desai and F. Rastegar. See item 2 above, pp. 351-360.

4. Geometric Modeling: A Status Report. M. R.. Corley. See item 2 above, pp. 467-474.

5. The Simulation of Heat Transfer in Castings and Weldments - Some Thoughts on Needed Research. P. N. Hansen and J. T. Berry. See item 2 above, pp. 497-502.

6. The Thermal Conductivity of Bentonite-Bonded Molding Sands. J. G. Hartley, D. Babcock, and J. T. Berry. Trans. AFS (89), 1981, pp. 469-476.

7. On Convection in Liquid Metal Molds. P. V. Desai and C. Kim. In Proc. of 2nd Intl. Conf. on Numerical Methods in Thermal Problems, Venice, Italy, 1981.

8. Initial Temperature Fields in Molds Prior to Solidification. P. V. Desai, C. Kim, and F. Rastegar. La Fonderie Belge (51), No. 3, 1981, pp. 3-5.

9. Foundries Are Closing the Computer Software Gap. N. G. Seman (Ed.), Foundry M&T, September and October 1981 (Parts I and II).

10. An Analysis of the Transient Edge Effect on Heat Conduction in Wedges. C. Wei and J. T. Berry. Intl. J. Heat and Mass Transfer (25), No. 4, 1982, pp. 590-592.

11. Extending the Modulus Approach to Feeding to Account for Corner Effects. C. Wei and J. T. Berry. Trans. AFS (90), 1982, pp. 193-200.

12. A Computer-Aided Design System for Castings. J. T. Berry, R. D. Pehlke and Associated Faculty Team Members. In Solidification Technology in the Foundry and Casthouse, 1982 Metals Society, London.

13. Geometric Modeling and Casting Solidification Simulation. J. A. M. Boulet and J. T. Berry. In CAD/CAM for Tooling and Forging Technology, Proc. of U.S./Sweden Workshop, Cornell University, Ithaca, NY. SME, Detroit, 1983.

14. The Q Method - A Compact Technique for Describing the Heat Flux Present at the Mold-Metal Interface in Solidification Problems. C. Wei, P. N. Hansen, and J. T. Berry. In Numerical Methods in Heat Transfer, Vol. II. Edited by R. W. Lewis, K. Morgan, and B. A. Schrefler. John Wiley and Sons, Ltd., 1983, pp. 461-472.

15. The Influence of Temperature, Moisture Content and Binder Content on the Thermal Conductivity of Dried Bentonite-Bonded Zircon and Silica Sands. J. G. Hartley and J. A. L. Patterson. Trans. AFS, 1983, pp. 183-190.

16. Solidification Simulation Based on the Edge Function Approach. C. Wei and J. T. Berry. Trans. AFS, 1983, pp. 509-514.

17. A CAD System for Solidification Simulation. J. A. M. Boulet and B. B. Dalton. Presented at the 1983 AFS Casting Congress (Paper No. 83-32).

18. Riser Design Using Edge Functions. C. Wei, J. T. Berry, and P. H. Franklin. In Proc. 1983 Engineering Foundation Conf. on Modeling of Casting and Welding Processes, 1984.

19. Software for Transient Heat Flow Simulation. M. J. Beffel, J. O. Wilkes, R. D. Pehlke, and J. T. Berry. See item 18.

20. Heat Losses in Runner Channels. P. V. Desai and C. W. Kim. See item 18.

21. The Application of Geometric Modeling to Metal Casting Technology. J. T. Berry and J. A. M. Boulet. In Proc. General Motors Symposium on Solid Modeling by Computers: From Theory to Applications. Warren, Michigan, September 25-27, 1983.

22. Computer-Aided Design for Castings. John T. Berry, R. D. Pehlke, and Associated Faculty Members. USA Official Exchange Paper, presented at 50th International Foundry Congress, Cairo, Egypt, November 6-11, 1983.

23. Mathematical Treatment of Numerical Solutions and Modeling in Solidification Simulation. John T. Berry and Robert D. Pehlke. Presented at CIATF Workshop, Cairo, Egypt, November 6-11, 1983.

24. Fictitious-Layer Method for Thermal Contact Problems. C. Kim and P. V. Desai, Numerical Heat Transfer (6), 1983, pp. 353-366.

25. Two-Dimensional Numerical Simulation of Casting Solidification with Heat Pipe Controlled Boundary Conditions. K. J. Wells, G. T. Colwell and J. T. Berry. Submitted for publication, Trans. AFS, 1984.

26. Computer Simulation of Forced and Natural Convection During Filling of a Casting, P. V. Desai, J. T. Berry, and C. Kim. Submitted for publication, Trans. AFS, 1984.

27. Moving Free Surface Heat Transfer Analysis by Continuously Deforming Finite Elements. C. Kim, P. V. Desai, and J. G. Hartley. Submitted to ASME Trans., March 1984.

28. Some Characteristics of the Conduction Heat Flux at the Surface of a Wedge Enclosure. C. Wei and J. T. Berry. Accepted for publication in ASME Journal of Heat Transfer.

29. A Theoretical Study of the Use of Heat Pipes in Metal Casting. G. T. Colwell, K. G. Wells, and J. T. Berry. To be published in Proceedings of the Fifth International Heat Pipe Conference, May 14-18, 1984, Tsukuba Science City, Japan.

TALKS GIVEN BY GEORGIA TECH TEAM MEMBERS

The following talks were given during the 1983 calendar year by various team members:

February	Savannah River Laboratories, Aiken, SC (Co-Director gave lecture)
April 11-15	Castings Congress, American Foundrymen's Society (two contributed papers given by team member Hartley and Project Director; see also <u>Trans. AFS</u>)
June 9	Sheffield University, England (seminar presented by Project Director)
June 11	Norwegian Foundrymen's Technical Association, Annual Conference, Frederikstad, Norway (Keynote Address, presented by Project Director)
June 21	Norwegian Foundrymen's Technical Association, Local Section, Jarlsø Støperi, Tønsberg (Seminar presented by Project Director)
June 23	Norwegian Foundrymen's Technical Association, Local Section, Sandnes (Seminar presented by Project Director)
June 23	Scandinavian Castings Research Commission (Støpforsk), Oslo, Norway (Keynote address by Project Director)
June 27	Technical University of Finland (Seminar presented by Project Director)
June 28	Royal Swedish Institute of Technology, Stockholm (Seminar presented by Project Director)
June 28	Royal Swedish Academy of Engineering Sciences, Stockholm Symposium on Computer Aided Design of Forgings and Castings (Keynote presentation by Project Director)
June 30	University of Surrey, England (Seminar presented by Project Director)
July 1	University of Wales, Swansea University College (Talk presented by Project Director)
August 1-5	Engineering Foundation Conference on Modeling of Castings and Weldments, Henniker, New Hampshire (Three papers presented by Director and Co-Director; Director also co-chaired Organization Conference)

August	Institute of Indian Foundrymen, Baroda, India (Co-Director gave seminar)
August	Faculty of Engineering, University of Baroda, India (Co-Director gave lecture)
September 26-27	General Motors Symposium on Geometric Modeling (Invited paper presented by team member Boulet and Project Director)
October 18	Advisory Board Meeting of CADCAST, Ann Arbor, Michigan (Presentation by team member Hartley and Project Director)
November 1	British Non-Ferrous Metals Technical Centre, near Oxford, England (Talk by Project Director)
November 3	Aachen Technische Hochschule, Giesserei Institut, West Germany (Talk by Project Director)
November 4	Technical University of (West) Berlin, Fraunhofer Institut (Seminar presented by Project Director)
November 8	50th International Foundry Congress, Cairo, Egypt (Official Exchange Paper for the United States presented by Directors of Georgia Tech and University of Michigan CADCAST Teams)
November 9	1st International Seminar on Solidification Simulation, Cairo, Egypt (Keynote paper presented by Directors of Georgia Tech and University of Michigan CADCAST Teams)
November 18	American Foundrymen's Society Piedmont Chapter, Regional Meeting, Atlanta, Georgia (Talk by Project Director)

I. DESIGN AND CONSTRUCTION OF A GEOMETRIC MODELER FOR CAD OF METAL CASTINGS

TASK I

INTRODUCTION

The present goal of Task I is to develop the ability to provide finite element/finite difference mesh databases in support of the solidification simulation software being developed by Prof. Wilkes at the University of Michigan (Task III). The primary accomplishments of the past year are

- installation of the TIPS-1'77 geometric modeling program,
- installation of the PADL-2 geometric modeling program, and
- acquisition of state-of-the-art graphics display hardware.

We are presently engaged in evaluating each of the two geometric modelers as to their potential usefulness in generating the desired meshes. We are also installing the new hardware and exploring ways of using it in conjunction with the geometric modeling software. We expect that by the end of 1984, we will have chosen one of the two modelers and begun development of enmeshment software.

ENMESHMENT SOFTWARE DEVELOPMENT

Wang et al. [1] summarize the desirable features of mesh-generation software and describe several techniques that could be used for automatic or semi-automatic enmeshment of planar regions. Desirable features include

- precise modeling of boundaries,
- local mesh refinement,
- minimal user input,
- no restrictions on mesh topology,
- mesh verification (appropriate connectivity, acceptable maximum aspect ratio, etc.), and
- bandwidth minimization.

In general, one would expect that any solid geometric modeler could be used to support three-dimensional (3-D) enmeshment software. However, the solid model itself is not what enmeshment software requires. Rather enmeshment schemes require information about the boundaries of the solid, so that they can be modeled precisely, and about the topology of the solid, so that a valid mesh can be achieved. The effort required to extract such information from a given modeler provides a measure of the modeler's suitability for support of enmeshment.

While some of the two-dimensional (2-D) enmeshment techniques described by Wang et al. [1] are, in principle, applicable to the problem of enmeshing a 3-D region, the problem itself is inherently complicated. It is not surprising, then, that inexpensive, ready-made 3-D enmeshment software is not available. Lacking such software, we refine the goal of Task I to be the development of a reasonably convenient 3-D enmeshment scheme that operates on a database generated by one of the two available geometric modelers. By "reasonably convenient", we mean specifically that the program/user interface should be friendly. (The degree to which the enmeshment technique is automatic is of little concern, since the initial requirement of the system is that it provide some small number of mesh databases in support of Task III.) The key ingredients of such an interface are: (a) software that is friendly by design and (b) sophisticated graphics hardware. The latter is essential if the user is to be called upon to make many decisions during enmeshment. Specific features that the device should have are: (a) high-resolution 3-D displays that can be rotated, translated, and scaled very easily, and (b) a light pen, or similar device, by means of which the user can easily interact with the program.

A survey of several 3-D enmeshment schemes is given by Wördenweber [2]. Various aspects of the problem have also been addressed by T. C. Woo at the University of Michigan, Ann Arbor, in unpublished research.

INSTALLATION OF TIPS-1'77

The TIPS-1'77 geometric modeler [3] has been installed on an IBM 4341 system belonging to the College of Engineering at Georgia Tech. The software obtained was designed to drive a Tektronix 4010 graphics terminal, none of which is available in Mechanical Engineering. Hence, the subroutines RWIND, VSINI, VSTERM, DEVICE, and GPSLTM were replaced by subroutine GOURI, which uses IBM GSP software to drive IBM 3250 (black and white) and IBM 3279 (color) graphics terminals. Although hard copy of the graphics screens is not yet available, installation of an IBM 4250 graphics printer is expected in the near future.

The installed version of TIPS-1'77 has the following capabilities:

- interactive solid geometric modeling using constructive solid geometry (CSG),

- sectioning of modeled objects,
- generation of a database representing the intersection of a modeled object with a grid of three sets of uniformly spaced planes, each set being normal to one of three global Cartesian axes,
- approximate calculation of "mass" properties,
- semi-automatic enmeshment of plane sections, and
- various rendering modes.

Although programs that interact with the TIPS-1'77 database to perform stress analyses and other functions are available from Cornell University, none of these has yet been acquired by Georgia Tech.

The installed version of TIPS-1'77 provides no topological information, although the topology of the modeled object is inherent in the model.

Boundary data are available in TIPS-1'77 only through the model/plane grid intersection database referred to above. In principle, the plane grid could be made dense enough so that boundary information would be as accurate as desired. To capture boundary features whose scale is much smaller than that of the entire modeled object would, however, be very expensive by this technique.

Figures I-1 and I-2 show examples of TIPS-1'77 renderings. Each curve is the intersection of the model with one of the grid planes referred to previously.

INSTALLATION OF PADL-2

The PADL-2 geometric modeler [4] is installed on a DEC VAX 11/750 computer belonging to the School of Mechanical Engineering. Although the software could be used with any of several graphics display devices, the only one of these presently available to us is the Tektronix 4010, as emulated by the Intecolor 2400. The School owns an HP-7221C plotter, which eventually will be used to make hard copies of PADL-2 renderings. This capability awaits the School's acquisition of appropriate graphics software by means of which the VAX could drive the plotter.

The installed version of PADL-2 supports the following activities:

- interactive solid geometric modeling using CSG,
- sectioning of modeled objects,
- generation of a database giving an accurate representation of the boundary of a modeled object, and
- various types of rendering.

Of the rendering modes supported by PADL-2, only the simplest (black and white, wireframe) is available on the Tektronix 4010. Because shaded color images cannot be rendered, much of the power of PADL-2 to communicate is presently unusable. Development of a software interface between the PADL-2 shaded image processor and the Tektronix 4027, a color device that can be emulated by the Intecolor 2400, is underway. However, because the Tektronix 4027 is not as powerful as other color devices, e.g., Lexidata, supported by PADL-2, even the successful completion of this effort may not yield acceptable renderings.

Like TIPS-1'77, PADL-2 does not provide topological information about a modeled object directly to the user, but could, in theory, do so because the topology is inherent in the model.

Accurate boundary data can be extracted directly from PADL-2 because the system contains software specifically designed to produce an accurate boundary representation.

GRAPHICS DISPLAY HARDWARE

As a result of a grant from the National Science Foundation and matching funds from the State of Georgia, the School of Mechanical Engineering has recently purchased a PS-300 graphics system manufactured by the Evans and Sutherland Corporation of Salt Lake City, Utah. This equipment allows the user to

- perform real-time rotation, translation, and scaling using manually-operated control dials,
- interact directly with computer via a cross-hair controlled by a stylus and pressure-sensitive pad,
- display color images with many thousand vectors, and
- perform quasi-dynamic hidden line removal.

Installation and testing of this equipment has only just begun. Eventually, the system will be used to display not only renderings produced by one or both of our geometric modelers, but also the 3-D meshes we plan to generate.

FURTHER WORK

The following activities will be pursued during the coming year:

- choosing between TIPS-1'77 and PADL-2 for further work,
- establishing the PS-300 as the display device for the chosen modeler,
- designing the enmeshment software to serve as interface between the chosen modeler and the software being developed under Task III.

REFERENCES

1. Wang, K. K., S. F. Shen, C. Cohen, C. A. Hieber, and A. I. Isayev, "Computer-Aided Design and Fabrication of Molds and Computer Control of Injection Molding," Progress Report No. 10, Injection Molding Project, College of Engineering, Cornell University, Ithaca, NY, 1984.

2. Wördenweber, B., "Volume-Triangulation," CAD Group Document No. 110, University of Cambridge Computer Laboratory, Cambridge, England, 1980.

3. Hashimoto, N., F. Lau, and K. K. Wang, "TIPS-1'77 Version, System Manual," Publication No. MME-01, Materials and Manufacturing Program, Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, New York, 1981.

4. Hartquist, E. E. and H. A. Marisa, "PADL-2 User's Manual", UM-10, Production Automation Project, College of Engineering and Applied Science, University of Rochester, New York, 1983.

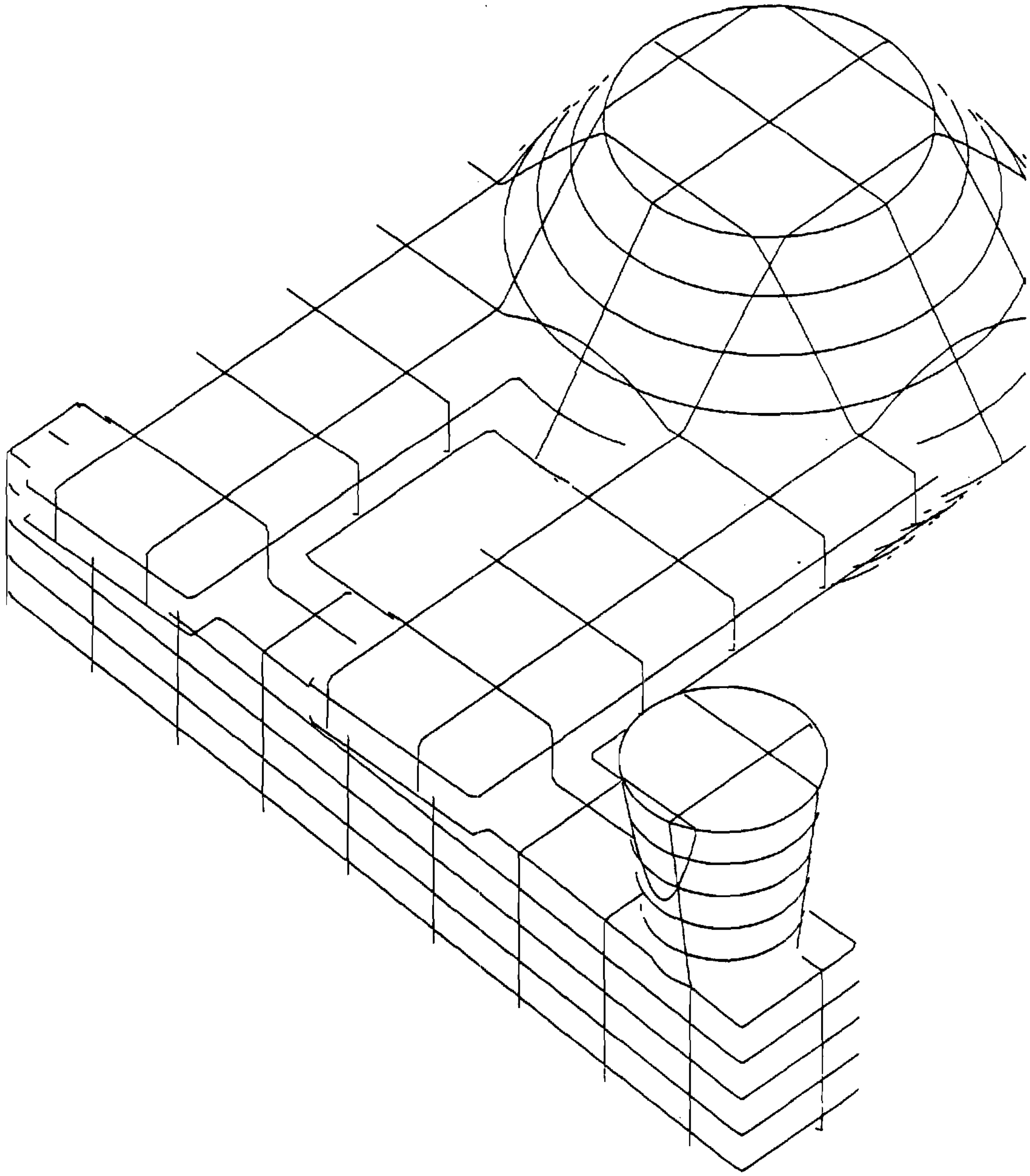

Figure I-1. A Casting and Gating System Modeled with TIPS-1'77.

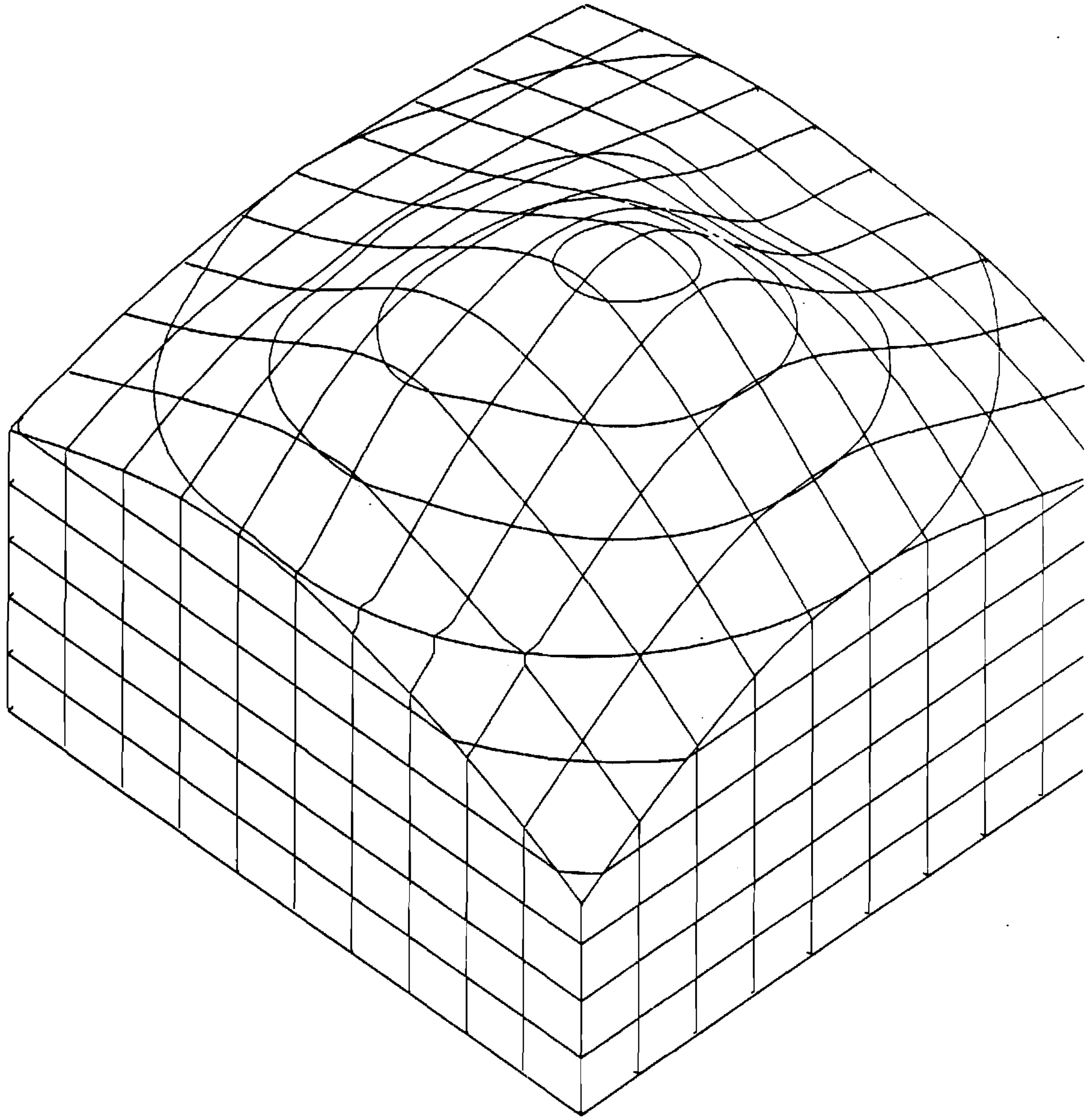

Figure I-2. Example of the "Elastic Face" Element in TIPS-1'77.

II. CHARACTERIZATION OF TRANSIENT EFFECT IN CURRENT AND FUTURE MOLD MEDIA

TASK II

INTRODUCTION

Any attempt to accurately model the solidification of a casting in a sand mold must include accurate predictions of the thermal properties of the mold medium. In molds composed of dry, bonded sand the thermal conductivity of the mold changes considerably with changes in temperature. While this has long been recognized, the recent work at Georgia Tech has also established the significant influence of the mold binder content and ramming density as well as the amount of moisture initially present in the mixture of sand and binder. Furthermore, the heat conducted away from the casting into a green-sand mold establishes large temperature gradients in the vicinity of the mold/metal interface. The moisture in the mold moves through the passages between sand grains, and the region of increased temperature near the casting drives moisture away from the hot metal interface. Since the metal temperature exceeds the boiling point of water, the water vaporizes and the moisture migration can become greatly accelerated causing portions of the mold to dry out completely.

The two areas of primary concern in Task II are the development of a predictive model for the thermal conductivity of bonded molding sands and the analysis of the heat and moisture transport in green sand molds. The predictive model is considered essential because determination of the influence of sand type, initial moisture content, binder content, ramming density and temperature on the effective thermal conductivity of bonded sands by experimental methods alone is an insurmountable task. The analysis of the transport mechanisms in green sands in important because the current state of knowledge is insufficient to predict or describe the increased chilling power observed when moisture is present in mold materials.

THERMAL CONDUCTIVITY MODEL ANALYTICAL MODEL DEVELOPMENT

Initial efforts related to model development for thermal conductivity were predicted on the assumption that the unit-cell model developed by Jackson [1980] could be modified for use with clay binders. However, a careful analysis of the model as well as comparison of model predictions and experimental measurements of the thermal conductivity of fluid-saturated sands (two-component systems) revealed that the unit-cell model does not adequately account for the effect of the volume fraction of sand. Furthermore, similar comparisons for a number of models available in the literature showed that none of the models were entirely adequate for predicting the thermal conductivity of two-component systems. This is illustrated in Figures II-1 and II-2.

Therefore, a new two-component model, called the cylinder-element model, has been developed under this grant to predict the thermal conductivity of fluid-saturated sands. As the geometrical characteristics of sands can be described statistically but not analytically, the influence of particle size, size distribution and shape are determined empirically for use with the cylinder-element model.

The effective thermal conductivity of a given sand type at room temperature depends upon the volume fraction of solid sand particles and the thermal conductivity of the saturating fluid. The model for thermal conductivity of such a two-component system, however, should be independent of the thermal conductivity of the saturating fluid.

A sand particle may have many points of contact with neighboring particles, and the number of contact points may vary with porosity or density. Since heat transfer will occur primarily by conduction through and near the contact points, the particle surface near a contact point can be considered to be a spherical surface. The corresponding geometrical model, called the cylinder-element model, is shown in Fig. II-3. The cylinder has radius r, a spherical surface of radius R, and a characteristic minimum separation distance δ.

In this model heat transfer through the direct contact is neglected because measured values of thermal conductivity of sands in vacuum are very much smaller than the values encountered with air-saturated sands. Heat transfer is assumed to flow only in the y-direction in Fig. II-3, and the effective fluid volume fraction (i.e. that portion of the saturating fluid in series with the cylinder), ϕ_{ef}, is assumed to be independent of the volume fraction of solid particles.

The details of the mathematical development of the model are not reported here but are the subject of a paper being prepared for publication in a technical journal. The results of the analysis yield the following expression for the thermal conductivity of two-component systems:

$$k_e = \frac{2(1+\delta)}{\frac{1}{k_s} - \frac{1}{k_f}} \{a \ln[\frac{\sqrt{R^2-r^2}+a}{R + a}] - \sqrt{R^2-r^2} + R\} + (1 - r^2)k_f \qquad \text{(II-1)}$$

where k_s and k_f are the thermal conductivity of the solid particles and the saturating fluid, respectively and

$$a = \frac{\frac{1 - R}{k_s} + \frac{\delta + R}{k_f}}{\frac{1}{k_s} - \frac{1}{k_f}} \qquad \text{(II-2)}$$

The dimensions in the cylinder-element model which must be determined are the parameters R, δ and r. These can be calculated from the volume fraction of solid particles, the effective fluid volume fraction and experimental measurements of the effective thermal conductivity. The volume fraction of solid particles, a known quantity, is related to the unknown parameters by

$$\phi_s = \frac{h}{3(1 + \delta)} (6R - 3Rh - 3h + 2h^2) \qquad \text{(II-3)}$$

where $$h = R - \sqrt{R^2 - r^2}$$

and the effective fluid volume fraction is given by

$$\phi_{ef} = \frac{1}{3(1 + \delta)} [3h(2R - h)(\delta + h) - h^2(3R - h)] \qquad \text{(II-4)}$$

The following procedure can be used to determine R, δ and r:

a. Measure the effective thermal conductivity of a sand saturated with air and saturated with water at a constant volume fraction ϕ_s .

b. Using Eq. II-1 with two different values of k_e and k_f and Eq. II-3 with ϕ_s known gives three equations to solve for R, δ and r (for one value of ϕ_s).

c. With R, δ and r known, ϕ_{ef} can be calculated from Eq. II-4. Note that ϕ_{ef} is assumed to be independent of ϕ_s .

d. Additional measurement of k_e at different values of ϕ_s can be used to determine R, δ and r at other values ϕ_s by repeatedly solving Eqs. II-1, II-3 and II-4 simultaneously.

The results obtained by this procedure for Ottawa sand (a round silica sand) and Masonry sand (an angular silica sand) are shown in Fig. II-4.

The model can be simplified somewhat by assuming that the parameter δ varies linearly with ϕ_S, and experimental results seem to justify doing so. If this assumption is made, step (d) above must be done only once so that ϕ_{ef} and the curves of δ and R for a sand can be determined with only three experimental mesurements of effective thermal conductivity. Therafter, the model can be used to predict the effective thermal conductivity of the sand for any volume fraction of solid particles (or any dry density) and any saturating fluid.

MODEL VERIFICATION AND PREDICTION

With the assumed linearity of the dependence of δ upon ϕ_S, only three experimental measurements are required for each sand before effective thermal conductivity predictions can be made. Experimental verification of the model has been achieved for the Ottawa silica sand and four saturating fluids covering a wide range of thermal conductivity (See Table II-1).

Measured thermal conductivities and those calculated using the cylinder-element model are compared in Table II-2. The percent deviation between measured and calculated values is shown in parentheses in this table.

MODEL EXTENSION FOR THREE-COMPONENT SYSTEMS

The cylinder-element model for the thermal conductivity of two-component systems cannot be used directly to predict the thermal conductivity of bonded sands. Bonded sands are actually three-component systems containing sand particles, air and a bonding material such as resin or bentonite clay. Thus current modeling efforts are directed toward modifying the cylinder-element model for use with bonded sands.

A thermal conductivity model for three-component systems requires a knowledge of the thermal conductivity of the bonding material. Measuring the thermal conductivity of a continuous bonding media such as resin is not difficult, but bentonite bonds form porous structures at and near the particle contact points. Thus, methods must be developed for determing the effective thermal conductivity of the porous bentonite. This problem is addressed in the next section.

EXPERIMENTAL PROGRAM

Refinement of Thermal Probe Design

The basic design of the thermal probe used for high-temperature thermal conductivity measurements is described in Progress Report No. 3, December, 1982. However, ceramic cement (rather than the graphite-based coating) is now used to fill the air spaces in the four-hole ceramic tube and the gaps between the ceramic tube and the stainless steel sleeve.

A numerical finite-difference model was used to examine several aspects of the probe design and sample size. The results from the numerical model indicated that the design is adequate, but that the inserted length of the probe should be greater than 10 cm and that the power input to the thermal probe should be selected such that the temperature rise of the probe is less than about 30^{o}C during the first ten minutes of the test.

Sample Preparation for Bonded Sands

Bentonite-bonded sands must be well mixed with the desired initial water content to permit even distribution of the clay particles around the sand grains. This can be achieved by mixing the moist, sand and bentonite mixture in a small mulling machine, but the mixture should be kept in a sealed container for at least 24 hours before preparation of the test specimen.

The specimen is compacted in 1-cm layers to the desired dry density using a hand ramming tool designed and constructed during the period of this report. This ramming tool is illustrated in Fig. II-5.

High-temperature thermal conductivity tests are made using the 4kVA box-type furnace described in previous progress reports.

<u>Validation of Cylinder-Element Model</u>

One goal of the experimental program was to verify the cylinder-element model for two-component systems. This effort included thermal conductivity measurements on a silica sand with round particles (Ottawa silica) and an angular silica sand (Masonry sand) at various densities and with several saturating fluids having vastly different thermal conductivities (air, water, transformer oil and ethylene glycol). These measurements were used to establish two geometrical parameters which describe the influence of particle size and shape.

The results obtained from the model development phase of Task II are described in the preceding section and can be summarized as follows:

1. The cylinder-element model is capable of predicting the thermal conductivity of two-component systems within about two percent of values determined experimentally.
2. The requisite geometrical parameters of a sand for use in the cylinder-element model for two-component systems can be determined from three experimental thermal conductivity measurements.
3. The cylinder-element model can also be used to evaluate the thermal conductivity of solid (sand) particles when this value is unknown.

4. The cylinder-element model can be extended for use with three-component systems, such as bentonite-bonded sands. However the density and thermal conductivity of the bentonite must be determined.

Density of Bentonite Particles

A hydrometer method was used to determine the density of bentonite particles. However, because bentonite reacts with water, a transformer oil was used for the hydrometer fluid. With this method, the particle densities for western bentonite and southern bentonite were found to be 2.23 g/cm^3 and 2.13 g/cm^3, respectively.

Density of Dried Bentonite

The bentonite in a bonded sand forms a porous bridge or structure at and near the contact points between sand grains and increases the area available for heat transfer by conduction. In addition, some of the bentonite coats the sand grains but does not enhance heat transfer. Thus it is important to know the density and the effective thermal conductivity of the porous structure formed by the dried bentonite.

Bentonite was mixed with water using various water-clay ratios and dried in a small oven. The density of the dried bentonite was measured by the wax immersion method (ASTM C914), and the results, summarized in Table II-3, show that the dry density does not vary significantly with water-clay ratio for the range of values studied. In most foundry applications, the water-clay ratio is greater than 0.5, and therefore it is reasonable to assumed that the dry density of the bentonite bond is independent of the water-clay ratio.

Thermal Conductivity of Dried Bentonite

Measuring the thermal conductivity of dense bentonite samples with a thermal probe would require a large sample which is extremely difficult to prepare owing to the tendency of the clay to crack upon drying. Instead, a single heater wire having a diameter of 0.008 in was used (hot-wire method). A small thermocouple wire (0.003 in diameter) was placed beside the heater wire.

Bentonite clay was mixed with water to obtain the desired water-clay ratio, and the mixture was compressed in a cylinder to remove excess water and air. Next the mixture was extruded through a semi-circular hole and the sample was dried. The flat surface of the sample was finished with a file and a groove was formed to accept the heater wire and thermocouple. Two such samples were used to form a cylindrical specimen, and the air gap was filled with a silicone heat-sink compound. The specimen was 2 cm in diameter and 10 cm long.

The measured value of the room-temperature thermal conductivity of the dried bentonite sample was 1.11 W/m-K. The density of the sample has not yet been measured, but it is significantly higher than the bonds formed by bentonite in bonded sands.

The next step is to determine the effective thermal conductivity of the porous structure formed by the bentonite bonds. Experimental and empirical procedures necessary to accomplish this task are presently being developed.

GREEN SAND TRANSPORT MODEL

The analysis of heat transport in green sands has also been initiated during this phase of work on Task II. An analytical model, based upon the conservation of mass, energy and momentum, has been developed. This model would be used predict the temperature, moisture and pressure distribution in a green-sand mold during the solidification of castings. As the mathematical details of the model are quite lengthy and are being refined further, the complete derivation and explanation of the model will be included in the next progress report.

SUMMARY AND RECOMMENDATIONS

The goal of Task II is to provide information regarding the thermal behavior of mold materials for use with the simulation of casting solidification. Current efforts are concentrated on the development of a predictive model for the thermal conductivity of bonded molding sands and the analysis of the heat and moisture transport in green sand molds.

The results obtained during the period of this report can be summarized as follows:

1. A new model for the thermal conductivity of two-component systems has been developed and verified experimentally. This model is semi-empirical in that three experimental measurements are required to determine model parameters which account for sand particle size, shape, and size distribution. Predicted thermal conductivity values for two-component systems obtained with this model are within about two percent of experimental values.
2. The two-component, thermal conductivity model can be extended for use with three-component systems (e.g. bonded sands), but the density and effective thermal conductivity of the bonding medium must be determined.
3. Improvements in the design of the high-temperature thermal conductivity probe and in sample preparation have been made.
4. Experimental measurements of the density of western and southern bentonite particles have been completed.

5. An experimental procedure for determing the density of the porous structure formed by bentonite bonds in bonded sands has been developed. With this method, the relationship between water-clay ratio and the density of the porous structure has been studied.
6. The thermal conductivity of dense, dry bentonite samples has been measured using the hot-wire method, and work has begun on developing the procedures necessary to determine the effective thermal conductivity of the porous structure formed by the bentonite bonds.
7. An analytical model for the heat and moisture transport in green-sand molds has been developed.

During the next phase of Task II, the development of the predictive model for thermal conductivity will continue. The cylinder-element model, valid for two-component systems, will be extended to predict the behavior of three-component systems and to account for the influence of temperature on the apparent thermal conductivity of bonded sands. In particular, a three-component system whose binder is bentonite clay will be examined in detail. Considerable experimental work is necessary to achieve this goal. In addition, a method for determining the effective thermal conductivity of the bentonite bond must be developed.

The analysis of heat and mass transport in green sands will also continue. Implementation of the recently developed model in a finite difference computer program is planned. In addition, experimental work, including an instrumented test section compacted with green sand is planned for construction. Experimental measurements of temperature and drying in green sands will be used to validate and refine the analytical model.

Table II - 1 Thermal Conductivity of Saturating Fluids

Fluid	Thermal Conductivity (W/m K) at $25^{o}C$
air	0.0254
transformer oil (982-68)	0.11
ethylene glycol	0.263
water	0.609

Table II - 2 Comparison of Measured and Calculated Thermal Conductivities of Fluid-Saturated Ottawa Sand.

Volume Fraction of Solid Particles	Effective Thermal Conductivity (W/m K) of Saturated Sands for Various Saturating Fluids							
	Air		Transformer Oil		Ethylene Glycol		Water	
	Measured	Calculated	Measured	Calculated	Measured	Calculated	Measured	Calculated
0.58	0.284	0.281 (-1.1)	-	- -		-	2.925	2.922 (-0.1)
0.60	0.304	0.304 (0.0)	1.047	1.206 (-2.0)	1.851	1.877 (1.4)	3.036	3.036 (0.0)
0.62	0.329	0.330 (0.3)	1.094	1.090 (-0.4)	1.939	1.970 (1.6)	3.153	3.154 (0.0)
0.64	0.358	0.360 (0.6)	1.147	1.158 (1.0)	2.002	2.067 (3.3)	3.276	3.276 (0.0)
0.66	0.392	0.392 (0.0)	-	-	-	-	3.406	3.399 (-0.2)

Table II - 3 Density of Dried Bentonite

Water-Clay Ratio	Dry Density (g/cm^3) (western)	Dry Density (g/cm^3) (southern)
0.5	1.98	1.91
0.75	1.97	1.91
1.0	1.95	1.95
2.0	1.91	1.94
average	1.93	1.93

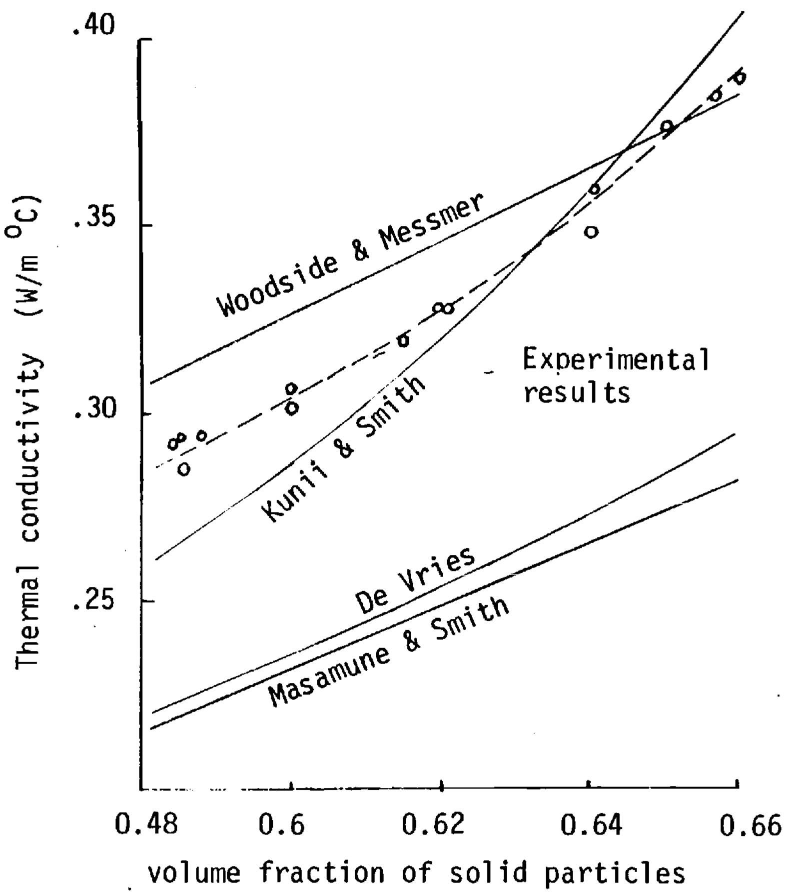

Fig. II-1. Comparison of 2-component models with experimental results for Ottawa sand saturated with air

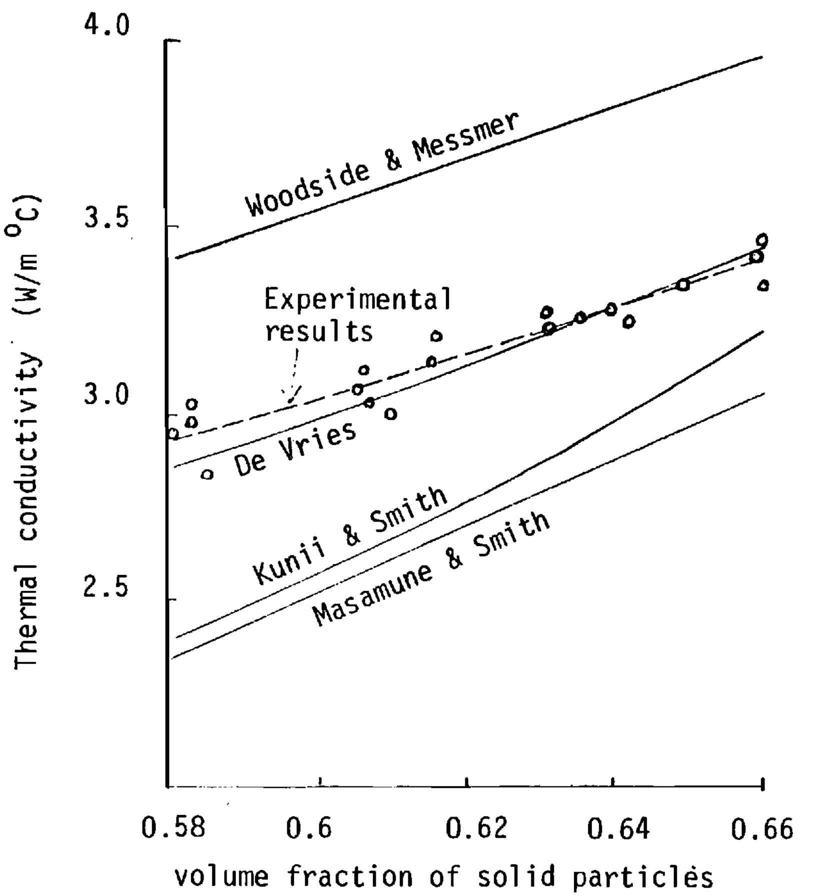

Fig. II-2. Comparison of 2-component models with experimental results for Ottawa sand saturated with water

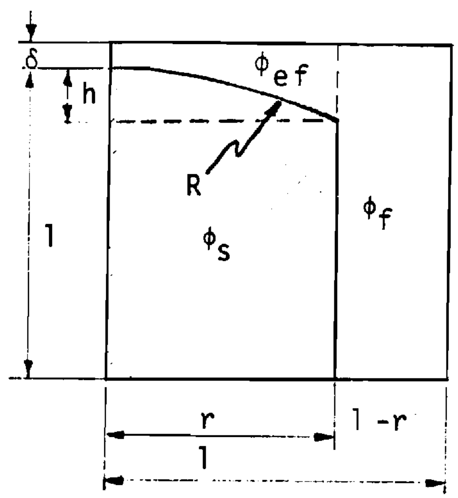

Fig. II-3. Cylinder-element model for two-component systems

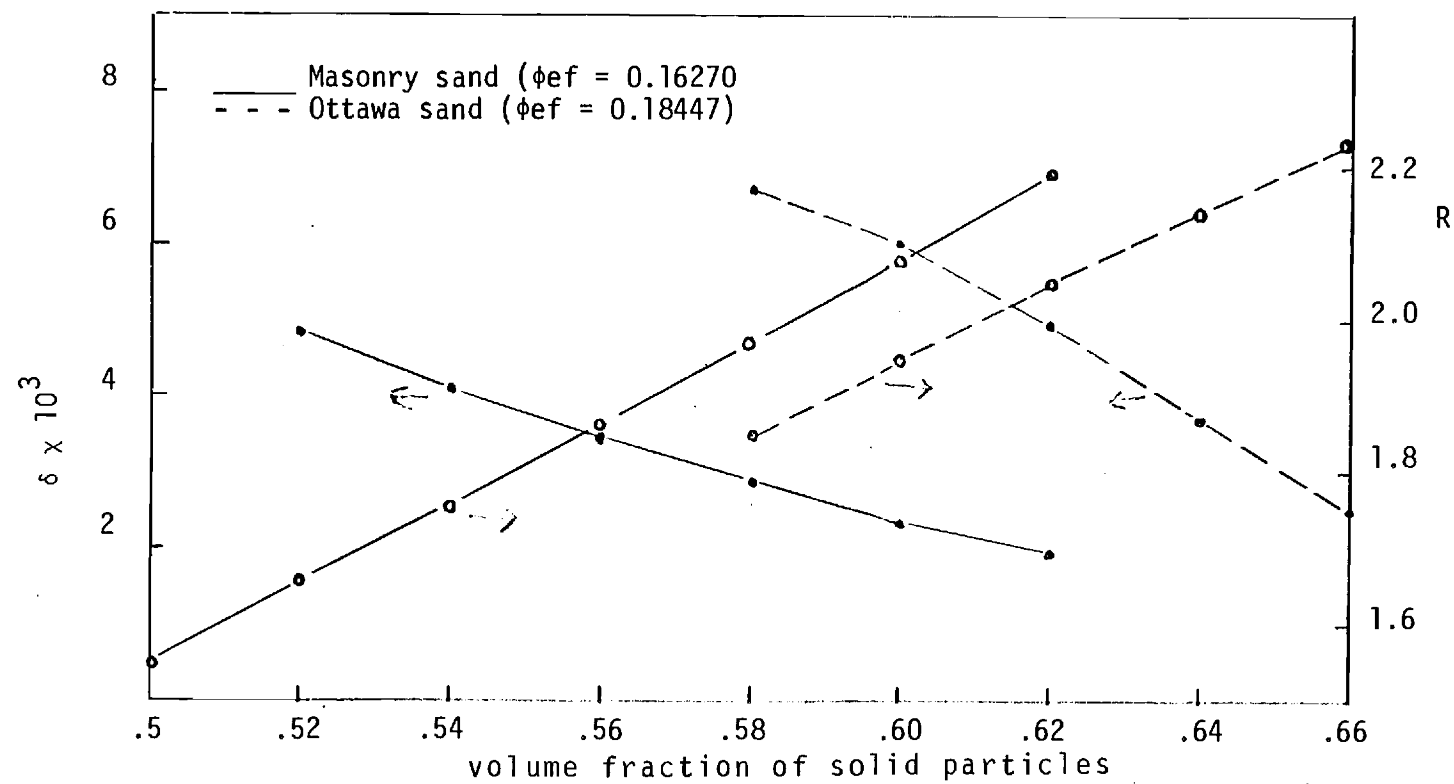

Fig. II-4. Variation of Parameters, R and δ for Ottawa Sand and Masonry Sand

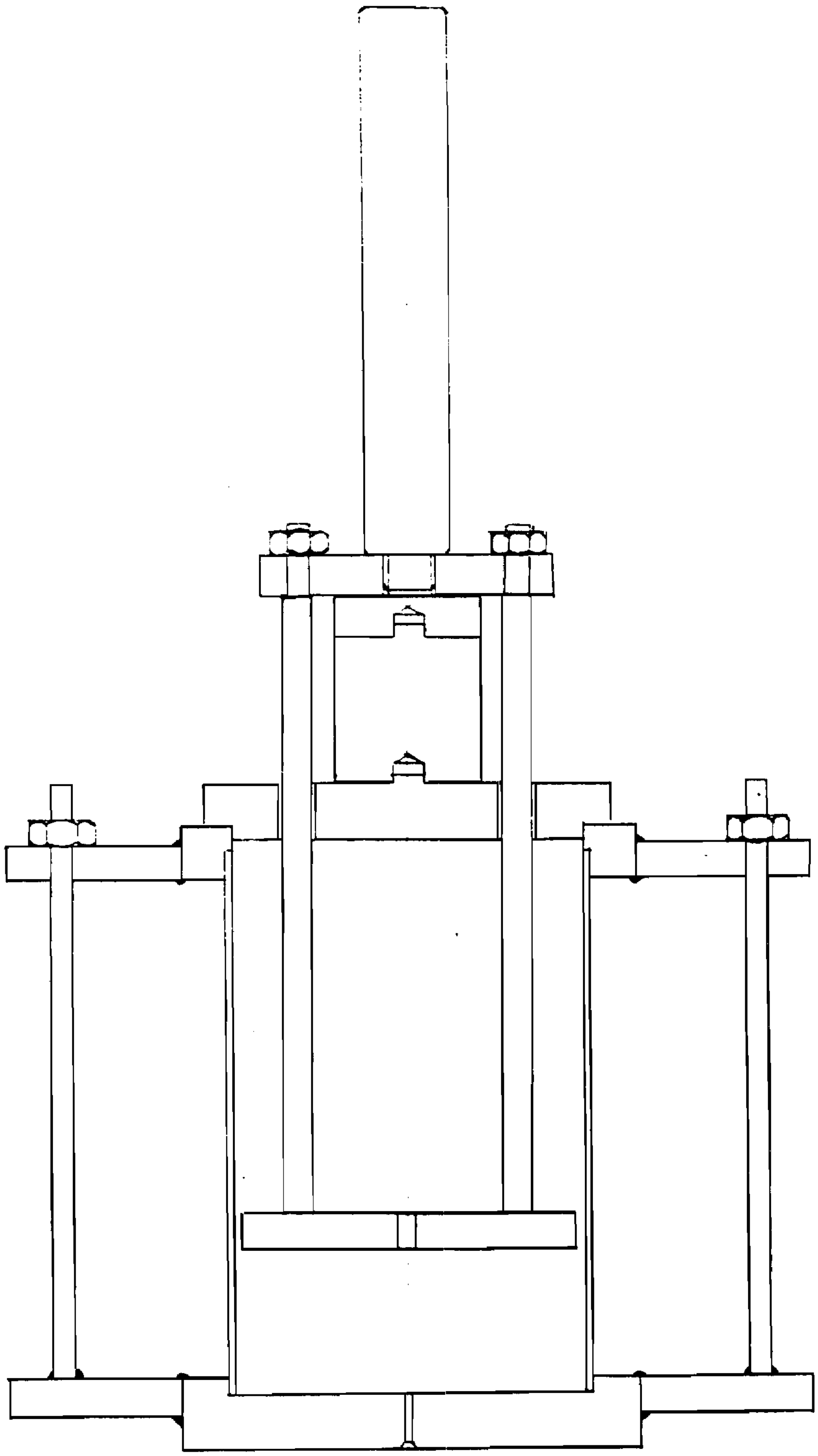

Fig. II-5. Configuration of Ramming Tool

IV. THERMAL CONVECTION DURING FILLING

TASK IV
THERMAL CONVECTION DURING FILLING

Overview of the Problem

The modeling of convection during filling of gating system was outlined in an earlier report [1] to include

a. the classical thermal contact problem between the mold and the metal,

b. the moving boundary problem due to the flowing liquid and

c. the moving free surface problem of the advancing liquid front in the gating channel.

It was also pointed out that the basic heat transfer process is a two phase (solid skiin-liquid metal), two media (mold-metal), moving thermal contact (flowing metal contacting the mold), moving free surface (of the liquid metal) problem that is further complicated by the geometry of the runners and the risers and the ingates and so forth.

In the research conducted under Task IV the problem has been viewed as one consisting of two main aspects. Only the first aspect concerning the loss of the liquid metal superheat as it flows with a free surface into and through an initially colder channel of mold material has been the focus of the research. The second aspect, concerning the sizing and locations of variations such as ingates, traps and risers and their influence on efficient unblocked flow performance of the runner system, must be considered a prime starting point for future research.

The moving boundary thermal contact problem was successfully solved by developing a novel finite element computation methodology and has been detailed elsewhere [1,2]. The present report provides the details on the advancing heat transfer calculations.

Advancing Free Surface Heat Transfer

Heat transfer to the initially cold empty mold channel from the advancing hot liquid metal requires a simultaneous calculation of the conjugated temperature fields within the mold and the metal. The mold/metal interface temperature is calculated by first writing the conservation statements for a control volume whose surface moves with an arbitrarily specified velocity field on it and then developing the corresponding finite element model. Such a formulation must include the effect of control volume deformation. For example, it has been shown [3] that for a fluid moving at a velocity, $\underset{\sim}{u}$, the energy equation written in a coordinate frame deforming at a velocity, $\underset{\sim}{u}^*$, takes the form

$$[\frac{\partial T}{\partial t} + (\underset{\sim}{u} - \underset{\sim}{u}^*) \circ \underset{\sim}{\nabla} T] = \alpha\nabla^2 T \ , \qquad \text{(IV-1)}$$

T being the fluid temperature. In writing this equation for a discretized finite element subdomain, $\Omega^e(t)$, a trial function, $\hat{T}$, which is an approximation of the unknown function, T, may be expressed by

$$T \simeq \hat{T} = [N]\ \{T\} = \sum_{j=1}^{p} N_j(x_i)T_j(t),\ (i = 1,2)\ , \qquad \text{(IV-2)}$$

where p is the number of nodes in each element and $n_j(x_i)$ are the time-independent shape functions. The conventional Galerkin procedure becomes applicable to the deforming control volume finite element analysis, without any need to assume the deformation to be temporarily stationary during each computational time step [4].

The interface temperature distribution due to forced convection to the instantly filled channel, discussed earlier [1,2] and shown in Figure IV-1 and

IV-2 is a special case of the general solution when mesh deformation vanishes. A comparison of results obtained by the fixed and the variable mesh formulations for the instantly filled case is shown in Figure IV-3. The close agreement seen in the results establishes a rationale to adapt the deforming control volume analysis for the moving free surface heat transfer problem. The specific case examined involves liquid aluminum flow at 100 cm/sec through a sand channel of semi-width 1 cm and a length of 20 cm.

The deformation pattern for the mesh, shown in Figure IV-4, consists of the element group (I) (10 layers normal to the flow direction) linearly expanding and the group (II) (6 layers normal to the flow direction) translating without deformation. The motion of the free surface is accommodated by varying the number of layers normal to the flow direction in the groups (I) and (II). In other words, at the beginning of the computation (when the free surface is about to enter the channel) the number of layers of group (I) is ten, and that of group (II) is six, as shown in Figure IV-4b. As the free surface reaches 2 cm into the channel the number of layers of group (I) is reduced to eight and that of the group (II) is increased to eight. This is shown in Figure IV-4c.

After the free surface reaches a distance of 10 cm from the channel entrance, only five layers are linearly expanding, as shown in Figure IV-4d. The reason for introducing such deformation pattern changes is the fact that reasonably small sizes of elements are used near the free surface region which have experienced a much more severe temperature gradients than those near the channel entrance region. The heat transfer coefficient, h, for the free surface, which must include the radiation effects, is assumed to be 100 $W/m^2 \cdot K$.

The interface temperature at the free surface, with the location of the free surface from the channel entrance denoted by y_{fs} (or time after the free surface enters the channel, t), is shown in Figure IV-5. The metal-mold interface temperature distributions from the channel entrance to the free surface at three different times (t = 0.02, and 0.01 and 0.2 sec) are also shown in this figure. The temperature distributions at the free surface corresponding to these three times are shown in Figure IV-6.d These figures show that the interface temperature at and near the free surface goes below the solidification temperature ($\theta \simeq 0.927$ in Figure IV-6). However, the following hot liquid raises this temperature. Therefore, it is inferred that freezing might have taken place along the wall and then part of this freezing zone might be remelted by the following hot liquid. It is interesting to note that when the free surface effects are not includes (the so-called instantaneous filling model), the lowest predicted dimensionlesss temperature for the liquid domain with U = 100 cm/sec is 0.9465, which corresponds to the thermal contact solution. Other researches [5] have pointed out that a solidified layer can form and partially remelt during initial filling in a sand mold casting.

The freezing and remelting process during the filling transient is a generally accepted phenomenon from the foundrymen's point of view. This is noteworthy, since in the special case of superalloys, such as those used in the precision castings of gas tubine blades, the mold wall is preheated to a temperature that will minimize freezing during the transient filling operation.

In Figure IV-5, the results at t = 0.2 sec obtained from the instantaneous filling model (otherwise the same as the advancing free surface heat transfer model) as also shown. There are significant differences between these results and those obtained from the moving free surface analysis at t =

0.2 sec when the free surface reaches the channel exit.

It is interesting to note that although the total heat transfer coefficient, h, for thefree surface was assumed to be possibly the largest reasonable value, the actual heat transfer from the free surface to the ambient (See Figure IV-6) is not significant as compared to that caused by the contact with the mold medium.

The results obtained when the free surface reaches the channel exit serve as an intial condition for the succeeding problem of a channel filled with liquid. Figure IV-7 shows the interface temperature distributions along such a filled channel at times t = 0.2, 0.3 and 0.4 sec. In Figure IV-8, the temperature distributions across the channel exit are shown. It is seen that the temperature drop near the channel exit recovers immediately after the free surface has passed through the channel.

Concluding Comments on Convection During Filling

Attempts to predict the patterns of solidification in shaped sand castings during the past two decades are based, also without exception, on an initial temperature field in the casting that is both uniform and instantly attained on pouring. A one-dimensional forced convection model to account for temperature losses in pouring has been used in the past [6] with an a priori assumption regarding the mold-metal interface heat transfer coefficient. Calculation of the latter involves a significantly more complex modeling of the type examined in this work.

The influence of convective effects is shown to be extremely important in shaped castings. Although the current findings remains to be validated by experiments, the concept of superficial solidification and remelting of the flowing metal stream would certainly seem to be borne out by earlier observations [7] regarding fluidity spirals. The formation of nuclei caused by chilling of the moving front as it enters through the gating system in the early stages of filling is not only consistent with these earlier observations, but is also germane to the problem of grain size control in superalloy precision casting. Convective effects of this type will also be of special importance in light section castings with extensive surface area, such as electronics-container boxes. In this instance not only will flow within the gating system be subject to such effects but the actual passage of the liquid metal within the mold-core assemly will experience forced convection. The estimation of temperature fields in castings of such complexity could be undertaken using similar techniques to those described in this work, together with experimental validation.

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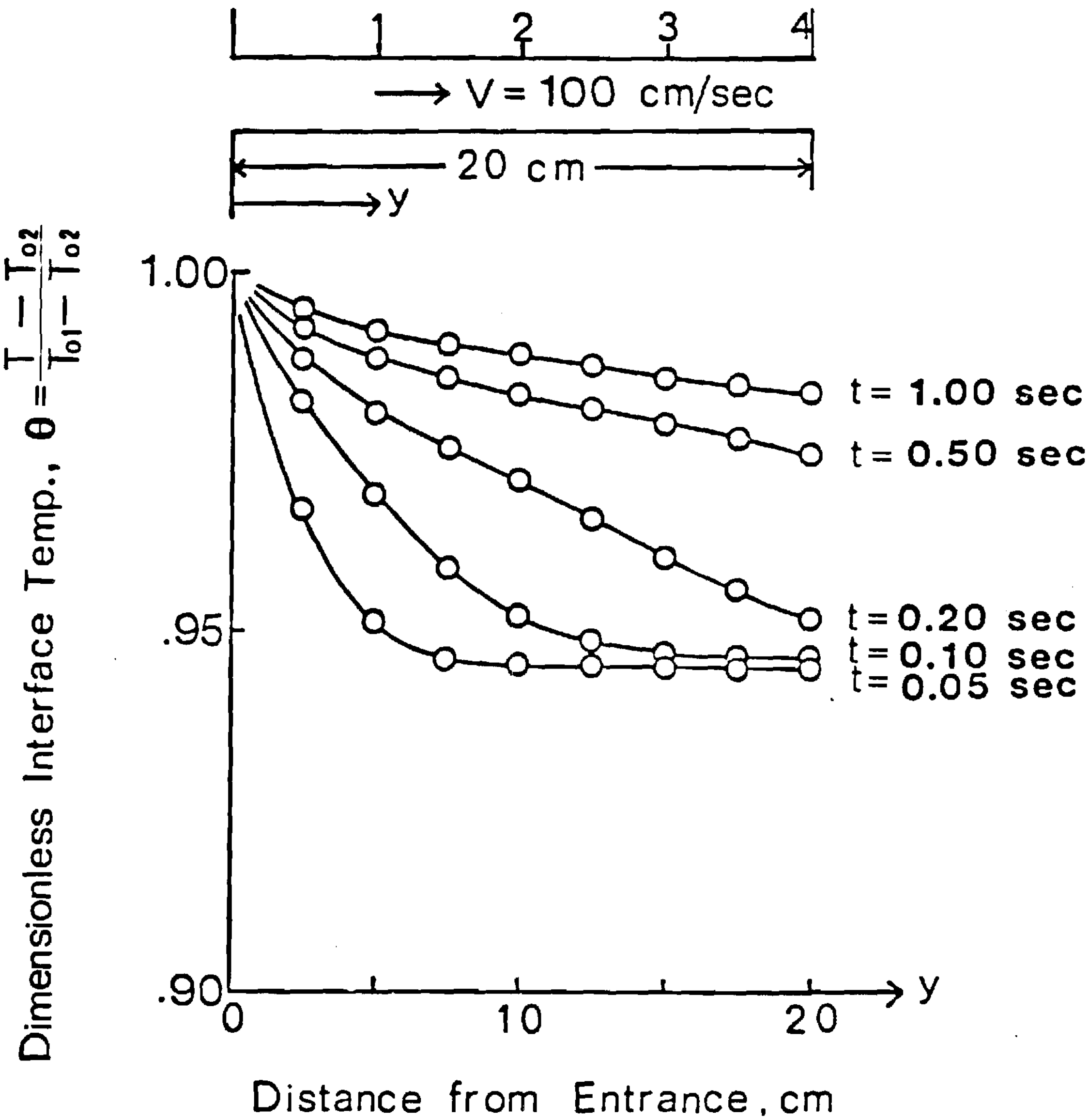

Figure IV-1. Mold-Metal Interface Temperature Variation along Channel at Different Times after Instantaneous Filling.

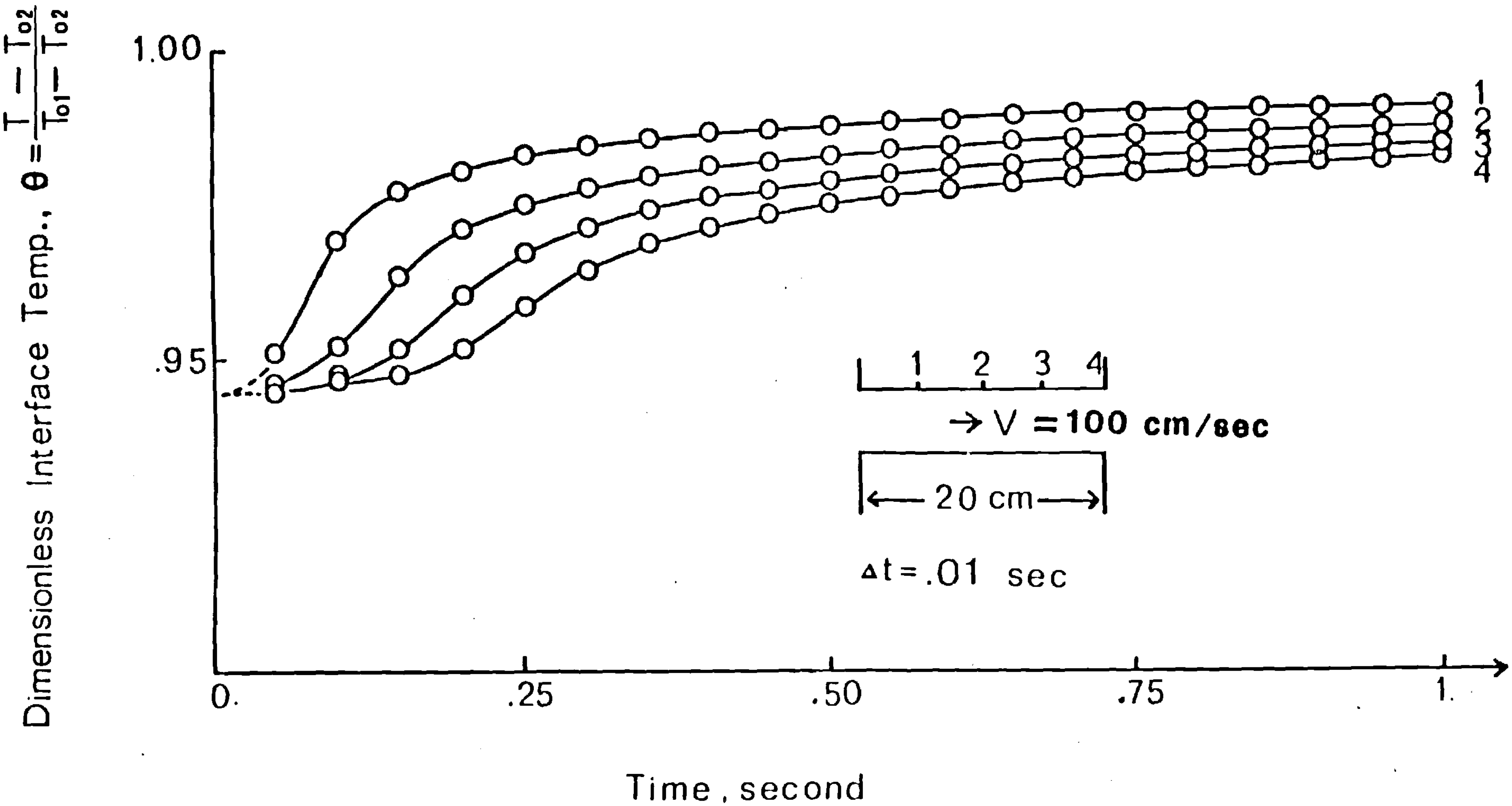

Figure IV-2. Temperature Responses at Various Locations along Mold-Metal Interface after Instantaneous Filling of Channel.

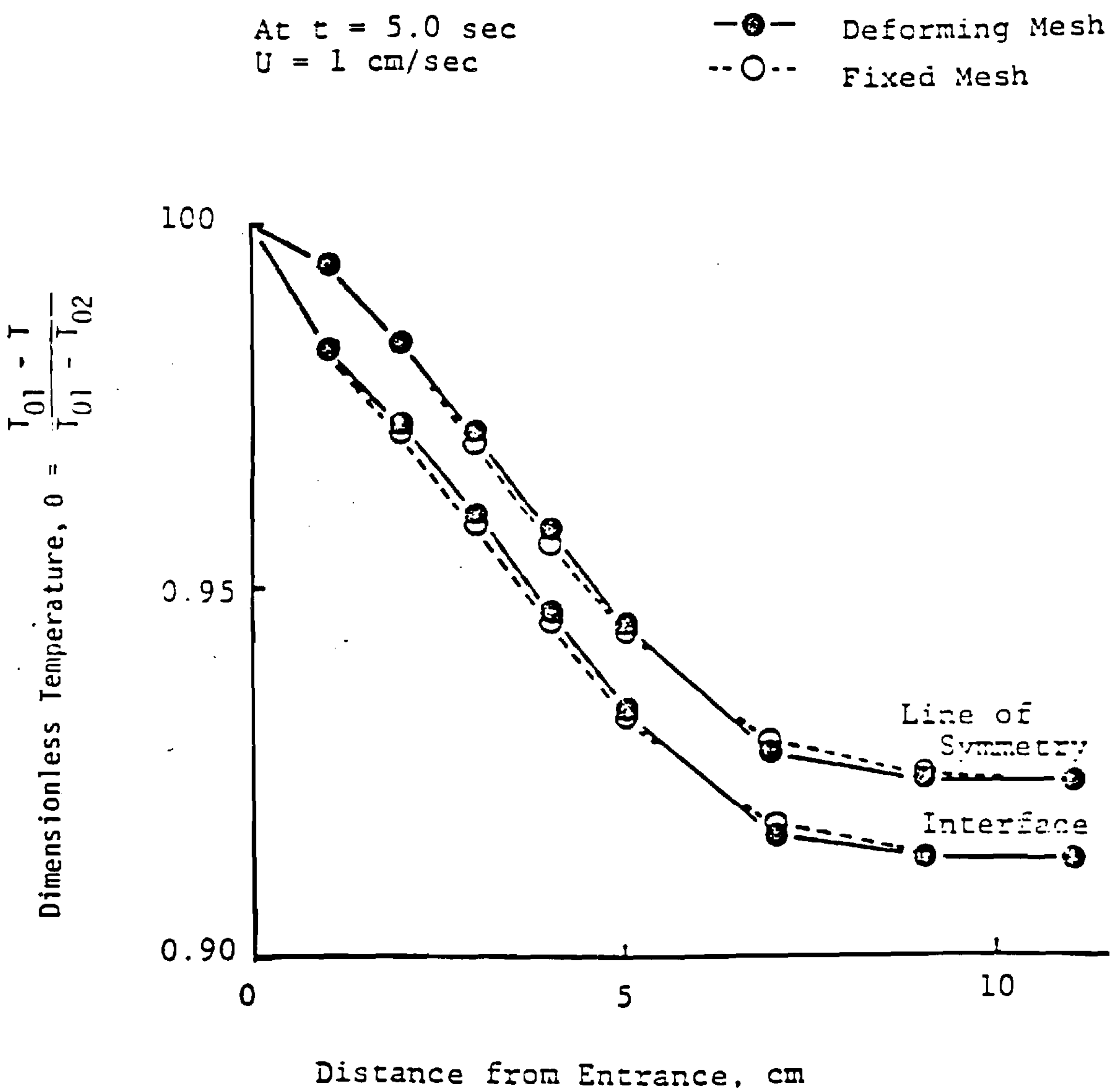

Figure IV-3. Temperature Variation along Channel Centerline of Symmetry and along Mold-Metal Interface Obtained by Two Different Numerical Schemes at 5 Seconds after Instantaneous Filling at Liquid Velocity of 1 cm/sec.

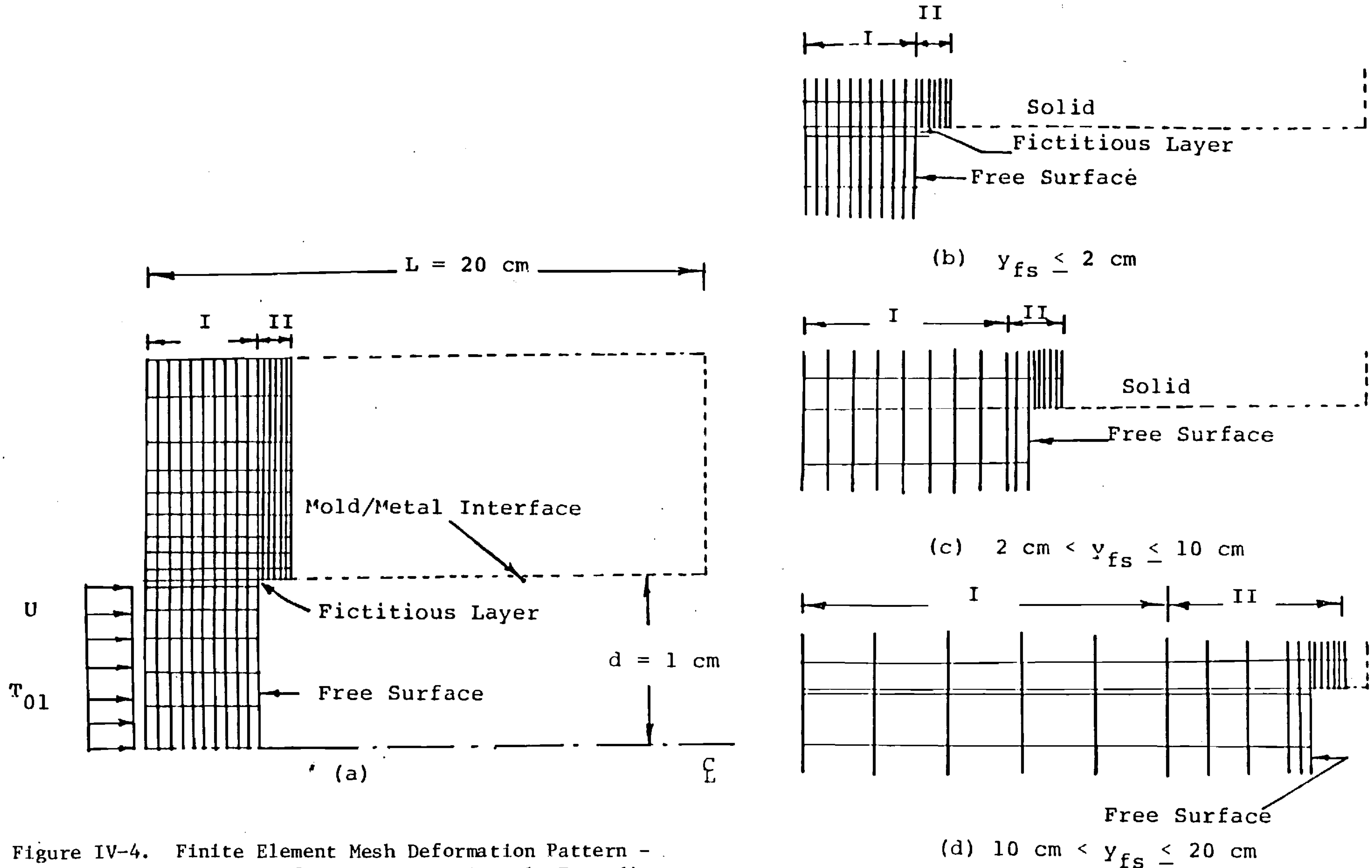

Figure IV-4. Finite Element Mesh Deformation Pattern - Elements of Group I are Linearly Expanding and Elements of Group II are Translating without Deformation. Figures (b) through (d) Show the Variation of the Mesh Pattern as the Free Surface of the Liquid Advances through the Channel.

y_{fs}: Location of the free surface measured from the channel entrance.

(I): Expansion, (II): Translation

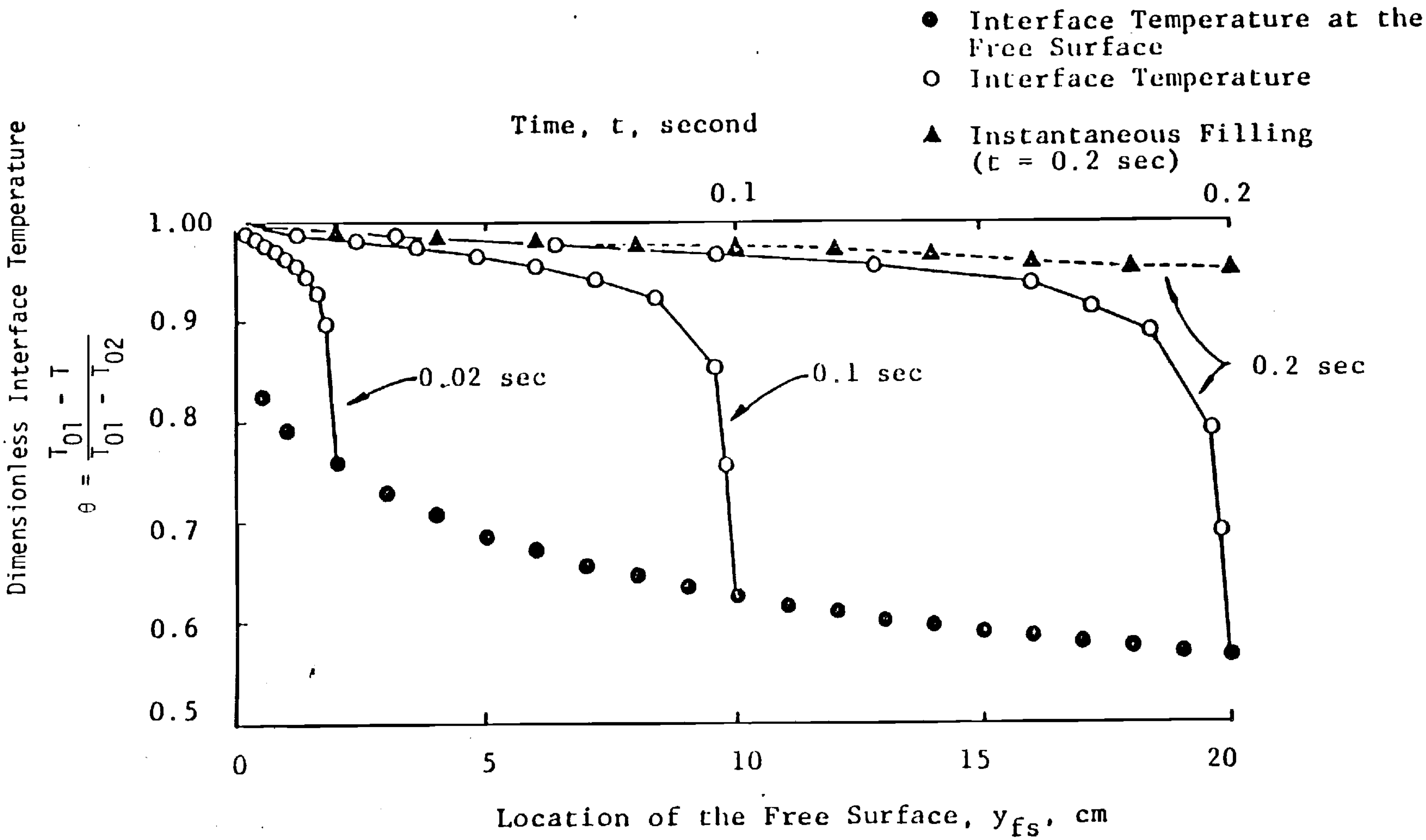

Figure IV-5. Mold-Metal Interface Temperature at Various Times as Metal Advances through Channel (Shown by Open Circles). For example, at 0.1 second the Free Surface of the Liquid Moving at 100 cm/sec has Advanced 10 cm into the Channel, Its Temperature Being Indicated by the Closed Circle. The Curve with Open Circles, Labeled 0.1 sec, Represents the Variation of Interface Temperature from the Channel Entrance to the Liquid Free Surface Location. The Filled Triangles Denote Temperature along Mold-Metal Interface at 0.2 second after Instantaneous Filling. The Dramatic Difference Between the Results for the Free Surface Temperature Obtained by Instant Filling Model vis-a-vis the Deforming Mesh Model is Noteworthy.

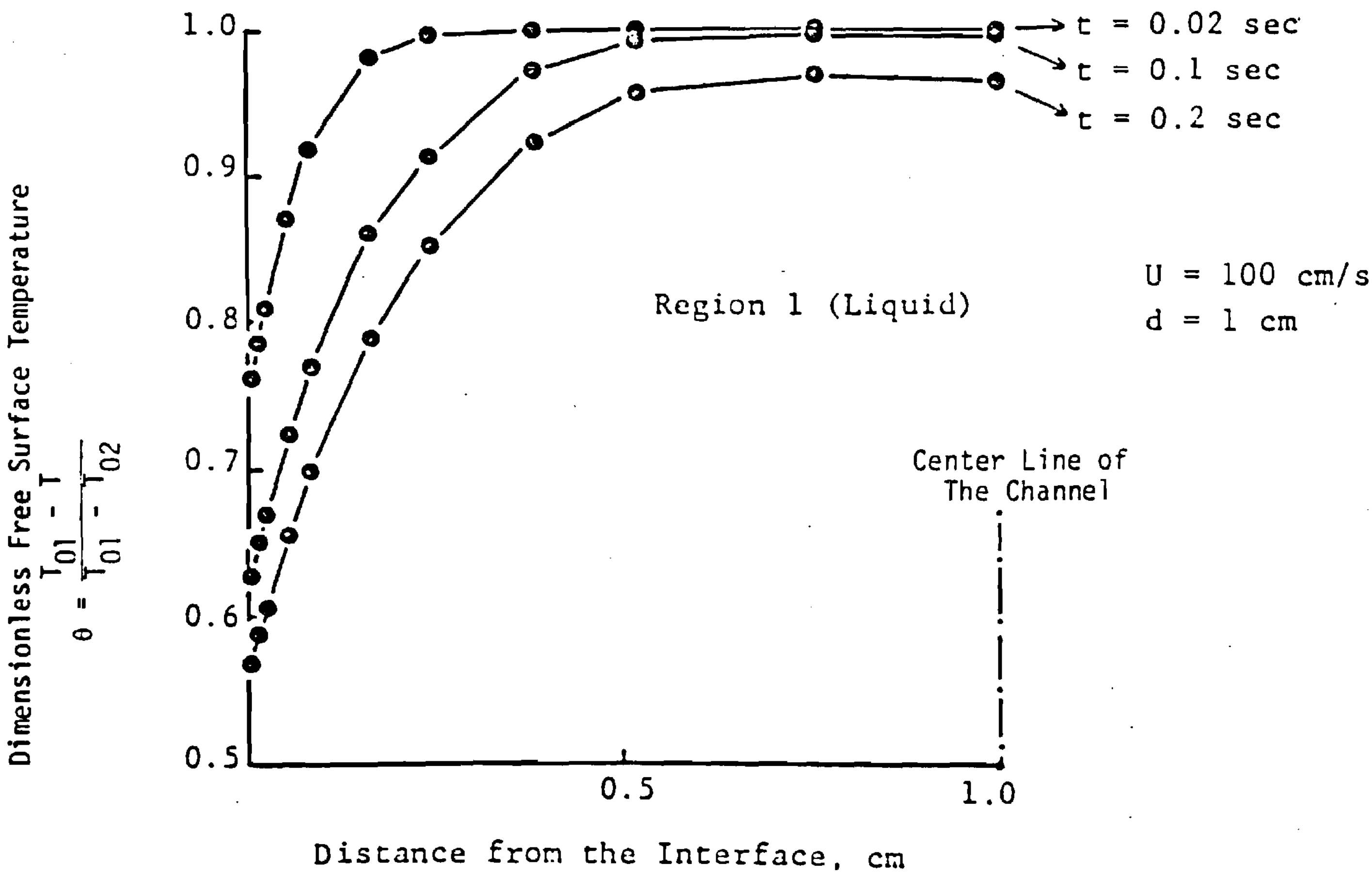

Figure IV-6. Temperature Distribution Across the Channel at the Free Surface Location at Various Times. Distance of 1 cm from the Mold-Metal Interface Indicates the Channel Centerline.

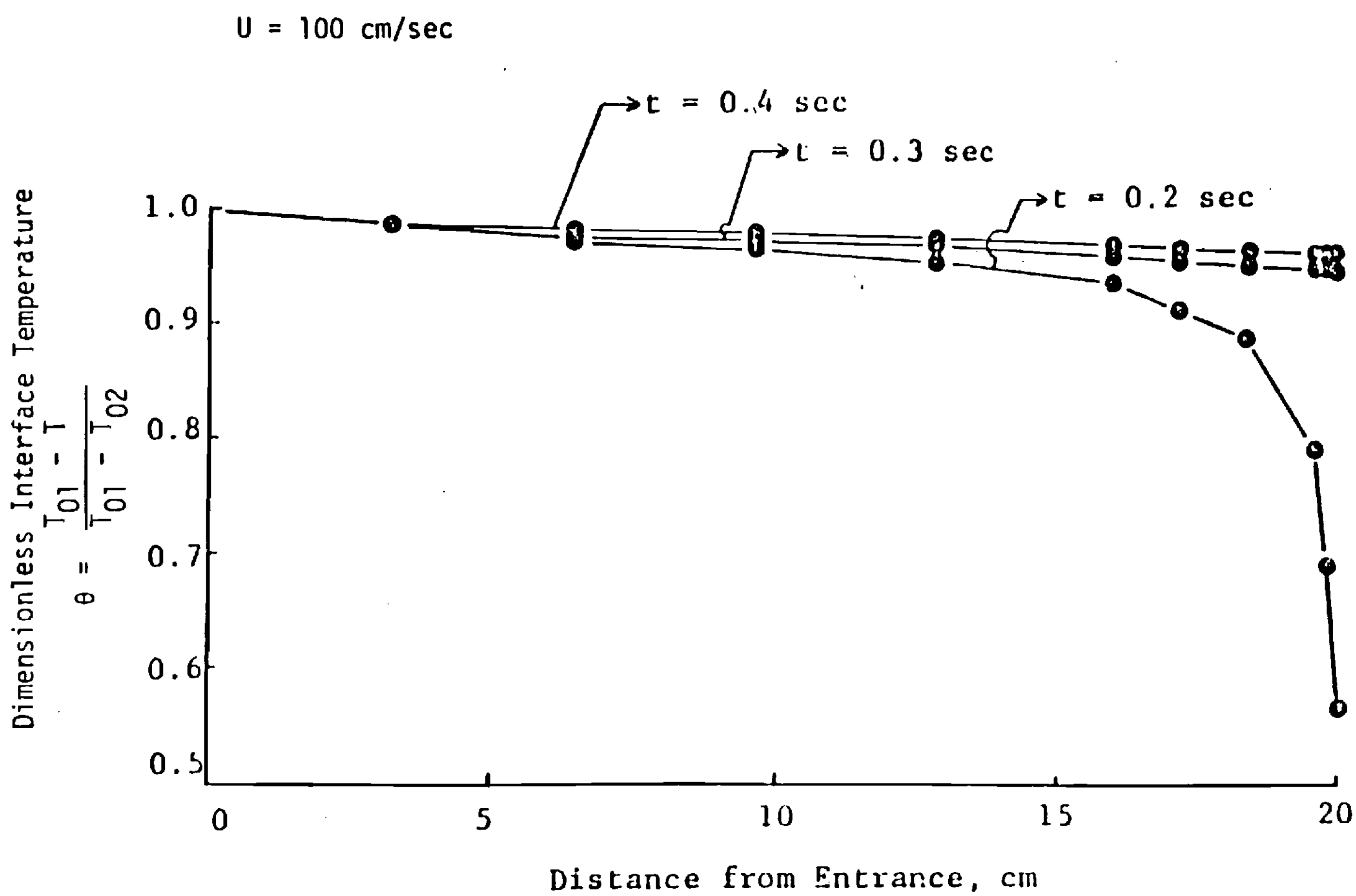

Figure IV-7. Mold-Metal Interface Temperature After the Free Surface Passage at 100 cm/sec through the Channel. At t = 0.2 second, the Free Surface Stands at the Channel Exit and the Curve, Labeled as 0.2 second, is Repeated from Figure IV-5 for Comparison Purposes. Interface Temperature for Subsequent Times Approaches the Instantaneous Filling Model Results.

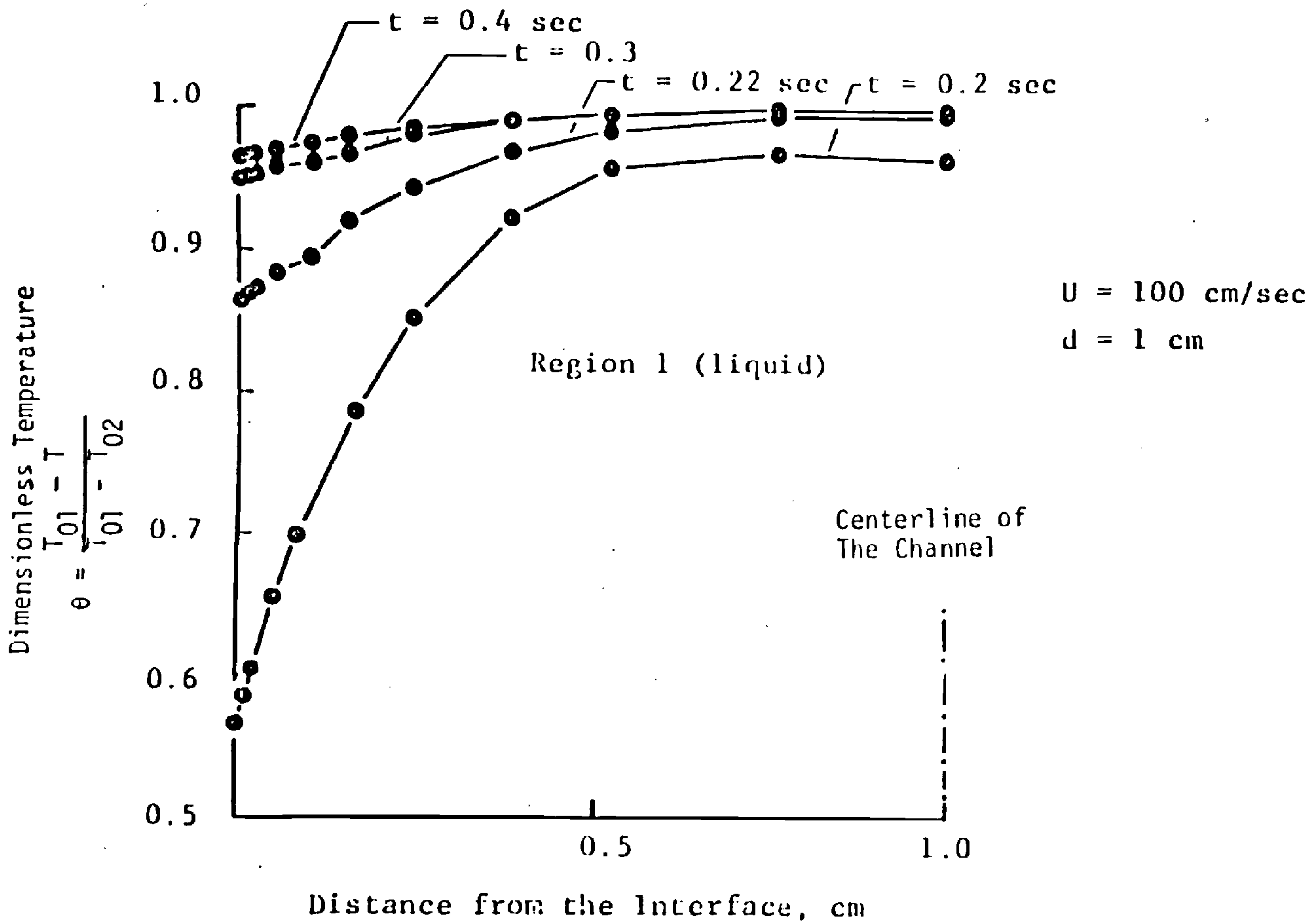

Figure IV-8. Liquid Metal Temperature Distribution across the Channel Exit on the Free Surface at Various Times.

VII. THE CONTROL AND PRESCRIPTION OF HEAT FLUX AT THE CASTING-MOLD INTERFACE

TASK VII

INTRODUCTION

Application of the principles of heat transfer to the study of solidification problems in various industrial processes has found moderate success in identifying and rationalizing the effects of important process variables. In the area of sand casting, the judicious dimensioning of feeders and the positioning of chills and insulating mold materials in order to achieve sequential solidification is, fundamentally, a problem of transient heat transport and coupled fluid flow. However, due to the complexities of casting geometries, temperature dependent thermal properties of mold and cast material and the nature of the mold metal interface, to name a few, analytical treatment of sand casting processes is only possible for the simplest of geometries and most idealized of conditions. Reliable models of sand casting solidification depend upon approximate numerical solutions to the transient heat conduction equation which are made possible by use of the digital computer. Numerical simulation schemes, unfortunately, are also limited in their general applicability to sand casting problems because of the relatively large computer capacity required to accurately model complex geometries and multicomponent alloy systems. Thus, analytical as well as numerical approaches to the modeling of sand casting processes must rely, in part, upon supplemental empirical and semi-empirical information.

Perhaps the most significant contribution to the rationalization of feeder head dimensioning was that due to Chvorinov [1], who related the solidification time of an infinite slab casting to its volume to surface area ratio or "modulus" as

$$t_f = \kappa\left(\frac{V}{A}\right)^2 \qquad \text{(VII-1)}$$

which is the well known Chvorinov's rule. In equation (VII-1) t_f is the solidification time, κ the so-called "mold constant" which depends upon the cast metal and the mold medium, and V and A are the casting volume and surface area respectively. Other workers have, since, developed modified versions of Chvorinov's one-dimensional rule, which take into account the two- and three-dimensional effects of finite casting geometry by the application of correction factors which account for curvature and corners [2-4]. It is this simple rule which provides the basis for quantitative feeder system design principles [5]. The Chvorinov approach was extended by Caine [6] who empirically accounted for the volume contraction of the cast metal due to solidification. Later, Adams and Taylor [7] developed an analytical expression which was similar to that of Caine. The work of Brandt, Bishop and Pellini [8] must also be cited here, as it helped to quantify the role of corners and cores in casting solidification. Furthermore, the work of Pellini [9,10] and Bishop, Myskowski and Pellini [11] was instrumental in establishing the role of temperature gradients and casting geometry on the feeding range of risers.

Recently, the problem of conduction of heat through a semi-infinite medium away from a boundary of arbitrary, two-dimensional angle held at constant temperature has received analytical treatment. Wei, in his Ph.D. thesis [12], was able to demonstrate a generalized invariability such that the difference between the rate of heat conduction per unit time and depth from any two-dimensional wedge and a corresponding planar area is invariant with time. He was, further, able to express this difference as a deviation integral,

$$\Delta^*(\theta_o) = 2\int_{-\infty}^{\infty} \cot\eta \left[\frac{1}{\pi \sinh\eta} - \frac{1}{\theta_o \sinh \frac{\pi\eta}{\theta_o}}\right] d\eta \qquad \text{(VII-2)}$$

which he termed the "wedge invariant" and which is a function of the wedge angle, θ_o, only. Wei used this result to extend the Chvorinov approach to two-dimensional geometries by assuming that the interaction of adjacent corners can be neglected and that the semi-infinite solution could be applied to a two-dimensional geometry by summing the effects of individual corners. In this way he characterizes the net divergence or convergence of heat flow away from a closed polygon with a constant interface temperature as

$$\mathring{Q}_{2d}(t) - \mathring{Q}_{1d}(t) = K_m(T_i - T_o)\left[-\sum_{n=1}^{N} \Delta^*(\theta_n)\right] \qquad \text{(VII-3)}$$

where K_m is the thermal conductivity of medium surrounding the polygon, T_i is the interface temperature, T_o the initial temperature of the medium and $\Delta^*(\theta_n)$ is the wedge invariant for the n^{th} corner of angle θ_n. Such a closed polygon and a tabulation of $\Delta^*(\theta_o)$ vs θ_o are shown in Figures VII-1 and VII-2 respectively. It follows from equation (VII-3) that the solidification time of such a polygonal shaped casting deviates from that given by a simple application of Chvorinov's rule and that this deviation is manifested by the opposing effects of diverging and converging heat flow away from external and re-entrant corners. A comparison of the ratio of the two-dimensional integrated heat transfer to the one-dimensional, Q_{2d}/Q_{1d}, versus

$1/\sqrt{t}$ as calculated from the wedge invariant and as obtained experimentally by Ruddle and Skinner [2] for a 6 cm square casting is shown in Figure VII-3. A 6 cm square casting was chosen here so that the corner area/total area ratio is unity according to Ruddle's definition, allowing direct comparison of his "corner correction factor" to Wei's "corner index". In light of the very approximate values obtained by Ruddle and the choice of average property data for dry silica sand, the values are felt to be in good agreement.

APPLICATION TO THE ANALYSIS OF RISER DESIGN

Inasmuch as the freezing time of a riser must be greater than that of a casting section which it is intended to feed, it follows that, where feed metal volume requirements are met, the overall riser volume required might be minimized by taking advantage of the converging heat flow effects of re-entrant corners. In this way, it might be possible to achieve improvements in casting yield which would justify the use of a more complicated riser design than the conventional cylindrical shaped feeder head. This concept has been applied commercially (Swedish Patent no. 8002279-1 B22 C 9/08) in the form of a cruciform cross-section as shown in Figure VII-4. It is possible, using the approach originated by Ruddle and Skinner [2] which was later extended by Wei et al [20], to compare the relative merits of cylindrical and cruciform shapes for use as risers.

The heat flux into a semi-infinite, solid medium away from a planar interface is given by [13]

$$\frac{\mathring{Q}_{1d}}{A} = (T_i - T_o) \frac{K_m}{\sqrt{\pi \alpha_m t}} \qquad \text{(VII-4)}$$

where α_m is the thermal diffusivity and A is the interface area. It follows from equation (VII-3) that

$$\mathring{Q}_{2d} = K_m(T_i - T_o)\left\{\left[-\sum_{n=1}^{N} \Delta^*(\theta_n)\right] + \frac{A}{\sqrt{\pi\alpha_m t}}\right\} \qquad \text{(VII-5)}$$

A similar expression can be written for a cylindrical interface which is valid for small values of time, t,

$$\mathring{Q}_{2d} = K_m(T_i - T_o)\left\{\frac{A}{2r_o} + \frac{A}{\sqrt{\pi\alpha_m t}}\right\} \qquad \text{(VII-6)}$$

where r_o is the cylinder radius. The approach alluded to earlier is to compare the heat flux from the plane wall with the time independent extra cooling effect due to the geometry. Examination of equations (VII-5) and (VII-6) shows that each expression embodies a term identical to the heat flux for the planar case and a time independent term.

Integration of equations (VII-5) and (VII-6) with respect to time yields the heat transfer in time, t, for the polygon and cylinder respectively,

$$Q_{2d} = K_m(T_i - T_o)\left\{\left[-\sum_{n=1}^{N} \Delta^*(\theta_n)\right] t + \frac{2A\sqrt{t}}{\sqrt{\pi\alpha_m}}\right\} \qquad \text{(VII-7)}$$

$$Q_{2d} = K_m(T_i - T_o)\left\{\frac{At}{2r_o} + \frac{2A\sqrt{t}}{\sqrt{\pi\alpha_m}}\right\}. \qquad \text{(VII-8)}$$

For the case of the cruciform,

$$-\sum_{n=1}^{N} \Delta^*(\theta_n) = -\left[4\Delta^*\left(\frac{\pi}{2}\right) + 8\Delta^*\left(\frac{3\pi}{2}\right)\right] = -0.4\,. \qquad \text{(VII-7a)}$$

If the cylinder radius, r_o, and the respective areas for the cylinder and cruciform are written in terms of their volume V and height h, equations (VII-9) and (VII-10) are obtained as follows:

$$Q_{CR} = K_m(T_i - T_o)\left\{\frac{24\sqrt{Vht}}{\sqrt{5\pi\alpha_m}} - 0.4\ ht\right\} \qquad \text{(VII-9)}$$

$$Q_{CY} = K_m(T_i - T_o)\left\{\frac{4\sqrt{Vht}}{\sqrt{\alpha_m}} + h\pi t\right\} \qquad \text{(VII-10)}$$

Taking the Chvorinov approach [1] and equating the heat lost from a cylindrical or cruciform riser in solidifying in time t_f to the integrated heat transfer in time t_f, the following are obtained:

$$\rho\lambda V = K_m(T_i - T_o)\left\{\frac{24\sqrt{Vht_{fcr}}}{\sqrt{5\pi\alpha_m}} - 0.4\ ht_{fcr}\right\} \qquad \text{(VII-11)}$$

$$\rho\lambda V = K_m(T_i - T_o)\left\{\frac{4\sqrt{Vht_{fcy}}}{\sqrt{\alpha_m}} + \pi ht_{fcy}\right\} \qquad \text{(VII-12)}$$

where $\rho\lambda$ is the volume latent heat of the cast metal. Equations (VII-11) and (VII-12) are quadratic in $\sqrt{t_f}$ and allow comparison of the freezing times for the two shapes. Here it is assumed that the superheat can be incorporated into the latent heat term and it must be noted that heat transfer from the ends is neglected. Equations (VII-11) and (VII-12) are plotted in Figure (VII-5) for a range of values of V and h for which the negligible end effects assumption is reasonable. From Figure VII-5 it appears that a longer freezing time as compared to the cylindrical riser cannot be achieved for the cruciform for the same volume of metal. This is due to the dominating effect of the increased surface area to volume ratio which cannot, in the simplified analysis, be overriden by the effects of the re-entrant corners.

ASSUMPTIONS AND LIMITATIONS

The validity of the analytical expressions developed to describe the corner effect rests upon the assumption of conditions not generally attained in foundry practice. Among these idealizations are: (1) the assumption of a constant interface temperature; (2) the assumption of negligible corner interaction; (3) the assumption of constant thermal properties; and (4) neglect of convection effects due to superheat of the molten metal.

With regard to the assumption of a constant interface temperature, it has been found to be a reasonable assumption for pure metals and certain geometries. However, it has been found to depend upon the size and geometry of the casting [3,21] as the attainment of a constant interface temperature depends upon the competing effects of conduction of heat away from the interface and conduction of heat to the interface due to the release of latent heat in the solidifying sections. It is clear from the data of Berry et al [3] and the numerical computations of Hansen [21], that the interface temperatures of cylinders and flat plate castings behave differently. It is, thus, reasonable to assert that the interface temperature for a solidifying cruciform might differ significantly from a solidifying cylinder. Indeed it is reasonable conjecture that the interface temperature may vary widely from point to point along the boundary of the cruciform.

The assumption of negligible corner interaction has been given some attention. Wei [12] has constructed a set of similarity curves which represent the ratio of the local heat flux at the boundary near a corner to the one-dimensional heat flux as a function of a dimensionless Fourier number

$$E_{\theta_o}(\eta) = \frac{\mathring{Q}(r,t)}{\mathring{Q}_{1d}(r,t)} \tag{VII-13}$$

$$\eta = \frac{r}{2\sqrt{\alpha_m t}} \tag{VII-14}$$

where r is the distance along the interface from the corner to a point on the interface. These similarity curves are shown in Figure VII-6. As can be seen from Figure VII-6, the corner effect is small for small time and large r for all except very acute angles. Franklin [15], in a series of experiments, recorded a time dependent behavior of the interfacial heat flux in agreement with the above observations. Clearly, it can be said that the assumption of negligible corner interaction depends upon the solidification time, and the proximity of the corners.

The assumption of constant thermal properties is clearly not valid. The experimental sand used in Franklin's study [15] (silica sand bonded with 5% bentonite plus 5% moisture, then dried) was found to have a thermal conductivity which decreased with increasing temperature in the range, 25-600°C. Thus it is possible that, for a re-entrant corner where heat flux converges, further biasing of the corner effect can occur owing to the local relative decrease in the thermal conductivity.

The effect of superheat has been studied briefly by Franklin [15]. He found that, once the superheat had been dissipated, the increase in heat flux for 60° and 90° wedges over the one-dimensional case appeared to be due to the edge effect alone. In a series of numerical experiments, Sciama [14] found that the effect of superheat was to increase the solidification time of various shapes while the relative solidification times remained unaffected. Sciama's computations, however, did not take into account convective currents which would arise and, in fact, be very different from shape to shape owing to the differences in geometry. While analysis of such phenomena can be quite complex, it is clear that such effects can be significant in solidification problems [16,17].

NUMERICAL MODEL FOR THE EXAMINATION OF BOUNDARY HEAT FLUX

A finite element model has been developed which can simulate the solidification of sand castings assuming uniform initial temperatures in the casting and mold. The model can take into account temperature dependent thermal properties of the mold and cast material and will allow latent heat evolution at constant temperature or over a temperature range. These characteristics make the model suited for studying the above corner and curvature effects on the heat transfer characteristics of molds with some relaxation of idealized conditions.

The model is a finite element formulation of the solution to the differential enthalpy equation first solved using finite differences by Sarjant and Slack [22] and used extensively by Hansen [21,23] in solidification studies. The finite difference formulation has been shown by Meyer [24], via a rigorous mathematical proof, to converge to the weak solution of the Stefan problem as the time step and node spacing approach zero. The enthalpy model has also been used by Shamsundar and Sparrow [25] in a finite difference, control volume formulation. Such a control volume formulation can be used in a finite element algorithm which would be equivalent to the so-called matrix method of Ohnaka and Fukusako [26]. However, the fact that elements which have circumcenters lying outside their boundaries cannot be used, except at convective boundaries, severely limits the applicability of this method.

For a pure metal at rest with density ρ, specific heat c, and thermal conductivity k, conservation of energy can be written

$$\frac{\partial E}{\partial t} = \nabla \cdot k \nabla T \qquad \text{(VII-15)}$$

where T is temperature, t is time and

$$E = \int_0^{T-T_s} (\rho c + \lambda \delta) dT \ . \qquad \text{(VII-16)}$$

In equation (VII-16) T_s is the liquidus temperature and λ is the latent heat of fusion with δ having the meaning that $\delta = 1$ at $T = T_s$ and $\delta = 0$ otherwise. A modified temperature scale is then defined as

$$\phi = \frac{C_s}{\lambda} \int_0^{T-T_s} \frac{k}{k_o} dT \qquad \text{(VII-17)}$$

where C_s is the specific heat at $T = T_s$ and k_o is the thermal conductivity at a reference temperature. Equation (VII-15) can then be written as

$$\frac{\partial H}{\partial Fo} = \nabla *^2 \phi \qquad \text{(VII-18)}$$

where

$$H = \int_0^{T-T_s} \left(\frac{C}{\lambda} + \delta\right) dT \qquad \text{(VII-19)}$$

$\nabla *^2$ is the dimensionless Laplacian operator, and $Fo = k_o t / \rho C_s L^2$. For the mold material

$$\frac{\rho m}{\rho} \frac{\partial H_m}{\partial Fo} = \frac{k_{om}}{k_o} \nabla *^2 \phi_m \qquad \text{(VII-20)}$$

where the subscript m denotes mold material and

$$H_m = \int_o^{T-T_s} \frac{C_m}{\lambda} \, dT \qquad \text{(VII-21)}$$

$$\phi_m = \frac{C_s}{\lambda} \int_o^{T-T_s} \frac{k_m}{k_{om}} \, dT \qquad \text{(VII-22)}$$

Equations (VII-18) and (VII-20) are the governing field equations for the mold and solidifying metal subject to the initial conditions

$$H = H_o \text{ in } \Omega \qquad \text{(VII-23)}$$

and boundary conditions

$$H = H_{S_1} \text{ on } S_1 \qquad \text{(VII-24)}$$

$$\nabla^*\phi \cdot \vec{n} + \overline{Bi} \, \frac{C_s}{\lambda} \, (\phi - \phi_\infty) \text{ on } S_2 \qquad \text{(VII-25)}$$

where

$$\overline{Bi} = Bi \, (T - T_\infty)/(\phi - \phi_\infty) \qquad \text{(VII-26)}$$

and

$$Bi = \frac{hL}{k_o} \qquad \text{(VII-27)}$$

h is the heat transfer coefficient acting at S_2.

In equations (VII-23)-(VII-27) the appropriate properties are chosen depending upon whether the boundary lies on the mold or metal. For an alloy solidifying over a range of temperatures, equation (VII-19) can be modified as

$$H = \int_o^{T-T_s} \left(\frac{C}{\lambda} + \frac{\partial f}{\partial T}\right) dT \qquad \text{(VII-28)}$$

where some appropriate relationship is used to determine the fraction solidified, f, as a function of T over the phase change interval.

The finite element formulation of equations (VII-18) and (VII-20) with (VII-23)-(VII-27) is derived using the standard Galerkin method procedure [27] with the interpolation functions being the same for H as for ϕ. This procedure yields the following global matrix equations:

$$-[[k_{Vm}] + [k_{S_{2m}}]]\{\phi_m\} = [C_m]\{\dot{H}_m\} - \{R_{S_{2m}}\} \qquad \text{(VII-29)}$$

$$-[[k_V] + [k_{S_2}]]\{\phi\} = [C]\{\dot{H}\} - \{R_{S_2}\} \qquad \text{(VII-30)}$$

The element matrices are

$$[k_V]^{(e)} = \int_{V^{(e)}} \nabla^*[N]^T \cdot \nabla^*[N]\,dV^{(e)} \qquad \text{(VII-31)}$$

$$[k_{Vm}]^{(e)} = \frac{k_{om}}{k_o} \int_{V^{(e)}} \nabla^*[N]^T \cdot \nabla^*[N]\,dV^{(e)} \qquad \text{(VII-32)}$$

$$[k_{S_2}]^{(e)} = \frac{C_s}{\lambda}\, \overline{Bi} \int_{S_2^{(e)}} [N]^T[N]\,dS_2^{(e)} \qquad \text{(VII-32)}$$

$$[k_{S_{2m}}]^{(e)} = \frac{C_s}{\lambda}\, \overline{Bi}_m \int_{S_2^{(e)}} [N]^T[N]\,dS_2^{(e)} \qquad \text{(VII-34)}$$

$$[C]^{(e)} = \int_{V^{(e)}} [N]^T[N]\,dV^{(e)} \qquad \text{(VII-35)}$$

$$[C_m]^{(e)} = \frac{\rho_m}{\rho} \int_{V^{(e)}} [N]^T[N]\,dV^{(e)} \qquad \text{(VII-36)}$$

$$\{R_{S_2}\}^{(e)} = \frac{C_s}{\lambda}\, \overline{Bi} \int_{S_2^{(e)}} [N]^T \phi_\infty\,dS_2^{(e)} \qquad \text{(VII-37)}$$

$$\{R_{S_{2m}}\}^{(e)} = \frac{C_s}{\lambda}\ \overline{Bi}_m \int_{S_2^{(e)}} [N]^T \phi_{m\infty}\, dS_2^{(e)} \qquad \text{(VII-38)}$$

In equations (VII-29) through (VII-38), $\{\phi\}$ and $\{\dot{H}\}$ are the column vectors of nodal modified temperatures and time derivative of nodal dimensionless enthalpies respectively. The matrices [N] are the interpolation functions and $V^{(e)}$ and $S_2^{(e)}$ denote the volume and S_2 surface of the element respectively. The time derivative may be replaced by

$$\{\dot{H}\}_\beta = \frac{\{H\}^{n+1} - \{H\}^n}{\Delta Fo} \qquad \text{(VII-39)}$$

and the modified nodal temperatures evaluated at time level $n\Delta Fo + \beta\Delta Fo$ by

$$\{\phi\}_\beta = \beta\{\phi\}^{n+1} + (1-\beta)\{\phi\}^n \qquad \text{(VII-40)}$$

The system of equations to be solved at each time step is then

$$\frac{[C]}{\Delta Fo}\{H\}^{n+1} = \frac{[C]}{\Delta Fo}\{H\}^n - \beta[k]\{\phi\}^{n+1} - (1-\beta)[k]\{\phi\}^n + \{R_{S_2}\}_\beta \qquad \text{(VII-41)}$$

For $\beta = 0$ the algorithm is fully explicit and the $\{\phi\}^n$ can be determined from the $\{H\}^n$. For $0 < \beta \leq 1$ iteration is required within a time step since the $\{\phi\}^{n+1}$ are not known until the $\{H\}^{n+1}$ are calculated.

It has been found, in several solutions, to be advantageous to lump the capacitance matrix, [C], in the solidifying metal in order to suppress anomalous transient response of nodes adjacent to nodes nearing their solidification enthalpy. That is, nodes adjacent to nodes which are approaching $H = 0$ in the transient response appear to gain enthalpy.

This is not the case if the terms in the capacitance matrices for the liquid metal region are summed for each row and "lumped" along the main diagonal. Further, should the capacitance matrices in the mold region also be lumped, considerable savings in computer time can be achieved because the system of equations is then uncoupled. Mass lumping is a common practice in finite element solutions and has been discussed critically in the literature [28,29,30].

PRELIMINARY RESULTS OF THE NUMERICAL ANALYSIS

Several constant property solutions have been generated with emphasis upon gaining experience with critical time steps and mesh fineness in the mold near the interface in order to accurately calculate interfacial temperature gradients. The approach taken has been to choose mold and metal dimensions so as to approximate semi-infinite mold conditions, applying insulated boundary conditions at the mold/atmosphere surface. Figures VII-8 and VII-9 show the dimensionless temperature gradient in the mold at the mold/metal interface for the one-dimensional constant property solution for the pure aluminum/silica sand system (properties given in Table 1) and the mesh shown in Figure VII-7. Figure VII-8 shows the solution with lumping the capacitance matrix in the metal only and Figure VII-9 for the fully lumped capacitance solution. As can be seen, the model is capable of accurately modeling interfacial temperature gradients and no loss of accuracy results from the fully lumped capacitance solution. The model predicts that the interfacial temperature will remain at or near the solidification temperature throughout the solidification period and, hence, the numerically calculated gradients compare well with the exact solution for constant interface temperature. The temperature profiles in the metal and mold at various times for the two solutions are shown in Figures VII-10 and VII-11.

Figure VII-12 shows a two-dimensional mesh, using linear, triangular elements for a finite element model of a cruciform shape. Owing to the symmetry of the shape only one-eighth of the geometry need be used while setting the normal gradient equal to zero on the lines of symmetry. Figure VII-13 shows some preliminary results for the aluminum/silica sand, constant property solution superimposed upon the analytical results of Wei [20]. Here, the fully lumped capacitance solution was used and the solution was terminated at a relatively short time because the two-dimensional case tends to become much more expensive in terms of computer time and it is felt that the expense is not justified when experimenting with mesh fineness and time step size. However, the preliminary results tend to indicate that the model can accurately predict temperature gradients at the interface. Since, for linear interpolation functions, the gradient is constant within an element, a continuous curve for $\dot{Q}_{2d}$ as given by the analytical solution cannot be obtained numerically for linear elements. Thus, the results plotted in Figure VII-13 were obtained via an averaging procedure in which the value of the normal gradient of ϕ at r/L equal to the midpoint between two interface nodes was taken as the average of the gradients in the two elements on the mold side sharing those nodes. This procedure yields results consistent with the analytical solution and thus may provide some basis for calculating equivalent heat transfer coefficients to be applied at the interface in order to implement the $\dot{Q}$ method in a finite element algorithm using linear elements.

FUTURE RESEARCH

The preliminary results indicate that the numerical model is capable of determining temperature gradients at the mold/metal interface in two-dimensional solidification problems. The model formulation should allow relaxation of some of the idealized conditions assumed in the analytical solution for two-dimensional corners. For example, the assumption of a constant interface temperature is not required because the temperature fields in the metal and mold are simultaneously calculated. Temperature dependent thermal properties can be used in both the metal and mold by using appropriate ϕ vs. T and H vs. T relationships. Although at this time only initial metal temperatures at the liquidus temperature have been assumed, it may be possible to achieve some indication of the effects of superheat by using higher initial temperatures and some approximate effective thermal conductivity for the superheated metal. Also, no form of corner interaction is implicitly assumed as the field equations are being approximately solved for the finite geometry. The finite element program which has been written will also be capable of solving axisymmetric problems and, thus, it will be possible to study corners with cylindrical symmetry which are commonly found in real casting and feeder head geometries.

In light of the foregoing discussions, it is evident that a great deal might be gained from further study of the transient corner effect as it relates to sand casting solidification and riser design. Consequently, a study will be commenced focusing upon a comparison of the interfacial heat flux characteristics of the three basic shapes, cylindrical, cruciform, and planar (flat plate), during solidification. Both numerical simulation and experimentally derived information about the effects of superheat, the behavior of the interface temperature, and the effects of temperature dependent thermal conductivity will be obtained. Additionally, the interaction of adjacent corners in comprising the overall corner effects and resulting heat transfer characteristics of contoured sections will be examined. Such information is important in determining the validity of analytical models for describing the heat flux present at the mold metal interface for use in solidification simulation schemes as has been proposed by Wei et al [20] and the use of more approximate methods which have been advanced by Dantzig et al [31].

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NOMENCLATURE

A	surface area
Bi	Biot number, $\frac{hL}{k_o}$
$\overline{Bi}$	modified Biot number, $Bi(T-T_\infty)/(\phi-\phi_\infty)$
c	heat capacity of cast material
c_S	heat capacity of cast material at liquidus temperature
E	enthalpy
$E_{\theta_o}(\eta)$	edge function for wedge angle θ_o, $\dot{Q}_{2d}/\dot{Q}_{1d}$
f	fraction solidified
Fo	fourier number of cast material, $k_o t/\rho c_S L^2$
ΔFo	dimensionless time step size
h	height in equations (VII-9)-(VII-12), heat transfer coefficient in equation (VII-27)
H	dimensionless enthalpy of cast material
H_o	initial dimensionless enthalpy of cast material
k	thermal conductivity of cast material
k_o	reference thermal conductivity of cast material
L	characteristic length
$\vec{n}$	unit normal vector to surface
N	number of corners in a closed polygon
Q_{CR}	integrated heat transfer for a cruciform
Q_{CY}	integrated heat transfer for a cylinder
$\dot{Q}_{2d}$	two-dimensional heat transfer rate
$\dot{Q}_{1d}$	one-dimensional heat transfer rate

r	distance along interface measured from a corner
r_o	radius of cylinder
S_1	specified temperature boundary surface
S_2	specified convection boundary surface
t	time
t_f	solidification time
T	temperature
T_o	initial temperature
T_i	interface temperature
T_S	liquidus temperature
T_∞	ambient temperature
V	volume
α	thermal diffusivity of cast material, $\frac{k_o}{\rho C_S}$
β	fraction of a time step at which FEM equations are solved
δ	unit delta function evaluated at liquidus temperature
κ	mold constant
λ	latent heat of fusion of cast material
η	similarity variable, $r/2\sqrt{\alpha_m t}$
ρ	density of cast material
ϕ	modified dimensionless temperature of cast material
θ_o	wedge angle
Ω	solution domain
$\Delta^*(\theta_o)$	wedge invariant for wedge angle θ

Subscripts

m denotes mold material

n corner index

Superscripts

n time step

(e) denotes element

T denotes transpose of matrix

Matrices

$[k_V]$ volume integral of stiffness matrix

$[k_{S_2}]$ surface integral of stiffness matrix

$[C]$ capacitance matrix

$\{H\}$ dimensionless nodal enthalpy vector

$\{\dot{H}\}$ time derivative of $\{H\}$

$\{\phi\}$ dimensionless nodal modified temperature vector

$\{N\}$ interpolation functions

$\{R_{S_2}\}$ vector due to surface integral

TABLE 1

TABLE OF PROPERTIES USED FOR CONSTANT PROPERTY SOLUTIONS

$\rho = 2.4\ g/cm^3$
$\rho_m = 1.52\ g/cm^3$
$C = C_S = 1.25\ J/gC$
$C_m = 1.005\ J/gC$
$k = k_o = 2.3\ W/cmC$
$k_m = k_{om} = 0.006\ W/cmC$
$\lambda = 401.8\ J/g$

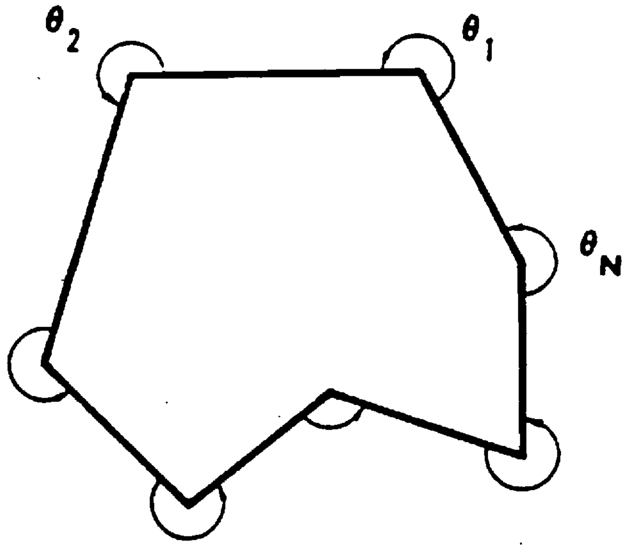

Figure VII-1. Polygonal Shaped Casting Cross-Section (after Wei [12]).

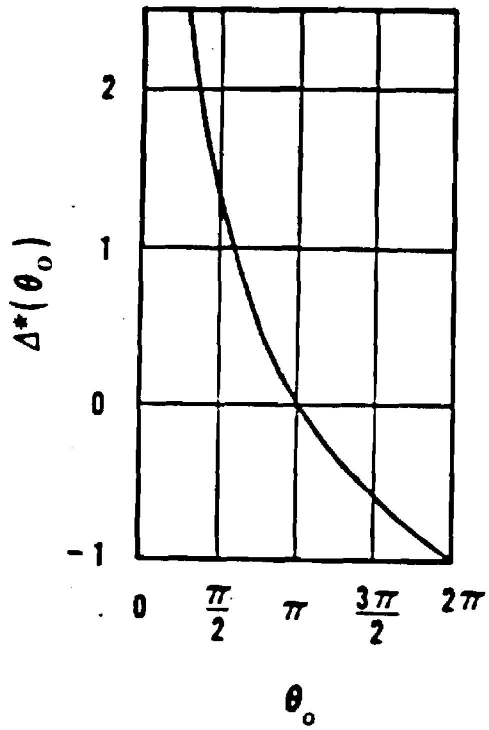

Figure VII-2. Plot of $\Delta^*(\theta_o)$ vs. θ_o (After Hurwitz and Roe [6]) (after Wei [12]).

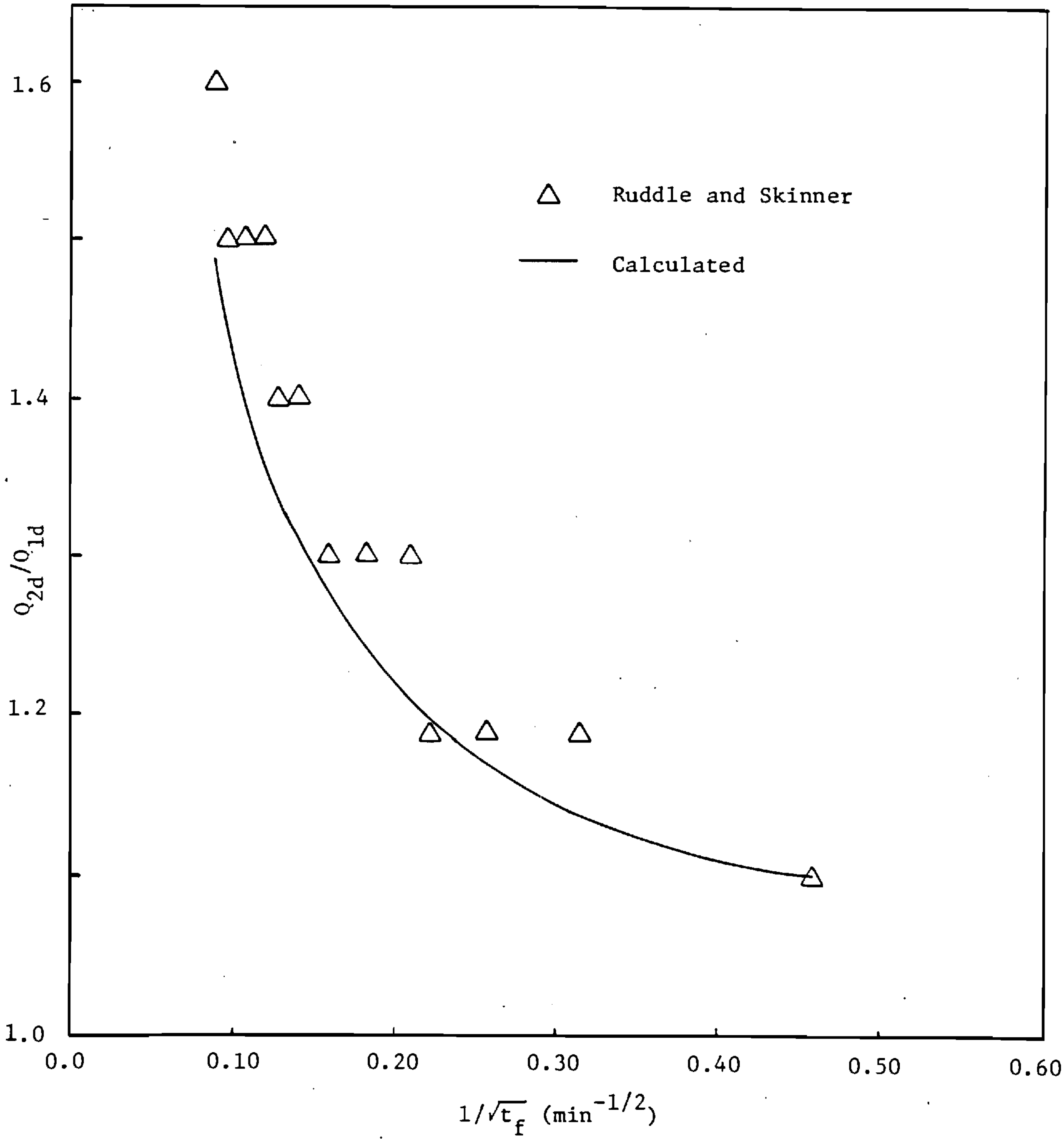

Figure VII-3. Plot of the Ratio Q_{2d}/Q_{1d} vs. $1/\sqrt{t}$ as Calculated from the Wedge Invariant and as Obtained Experimentally by Ruddle and Skinner [2] for a 6 cm Square Casting.

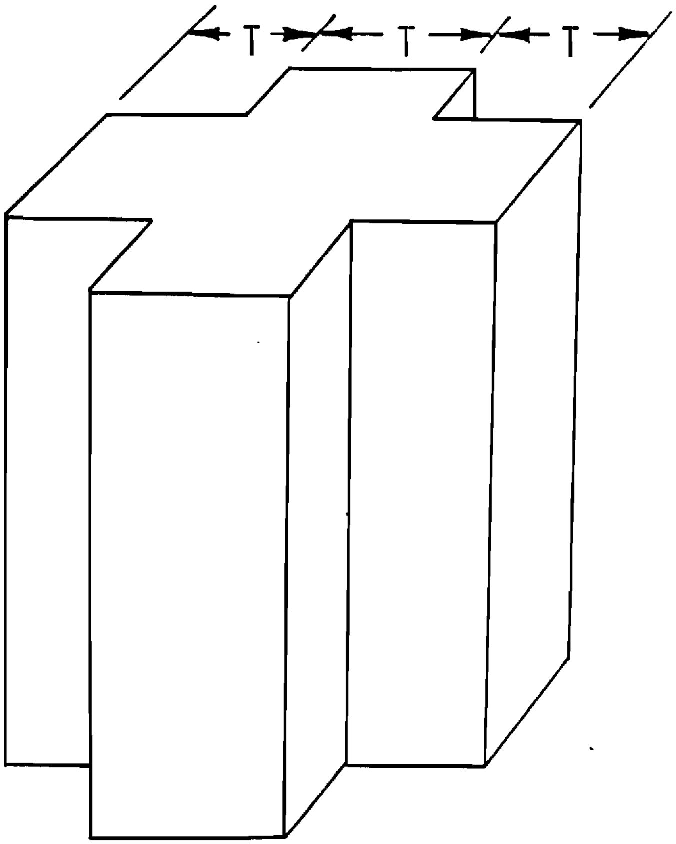

Figure VII-4. Cruciform Shaped Riser.

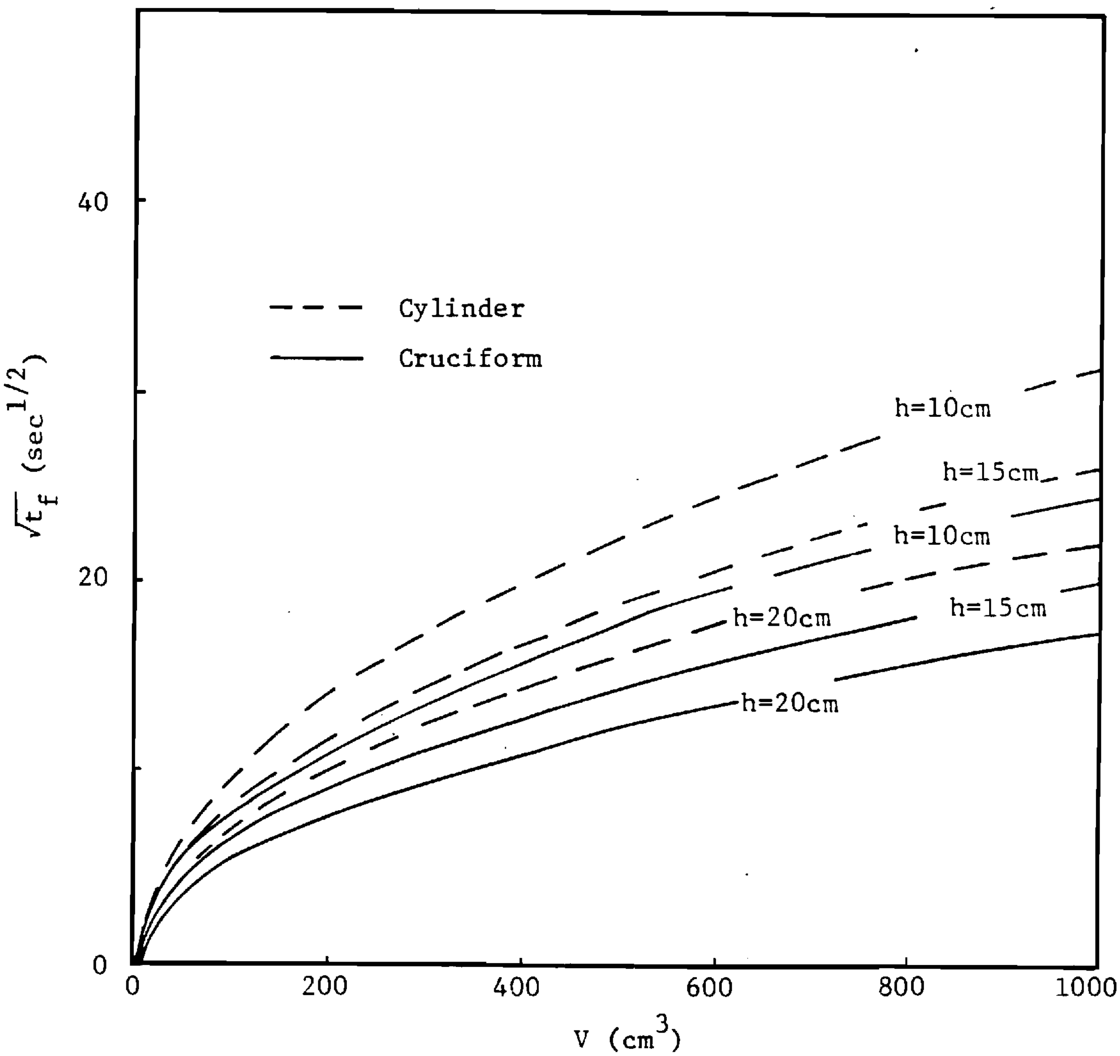

Figure VII-5. Plot of $\sqrt{t_f}$ vs. V with h as a Parameter for the Cylinder and Cruciform Risers.

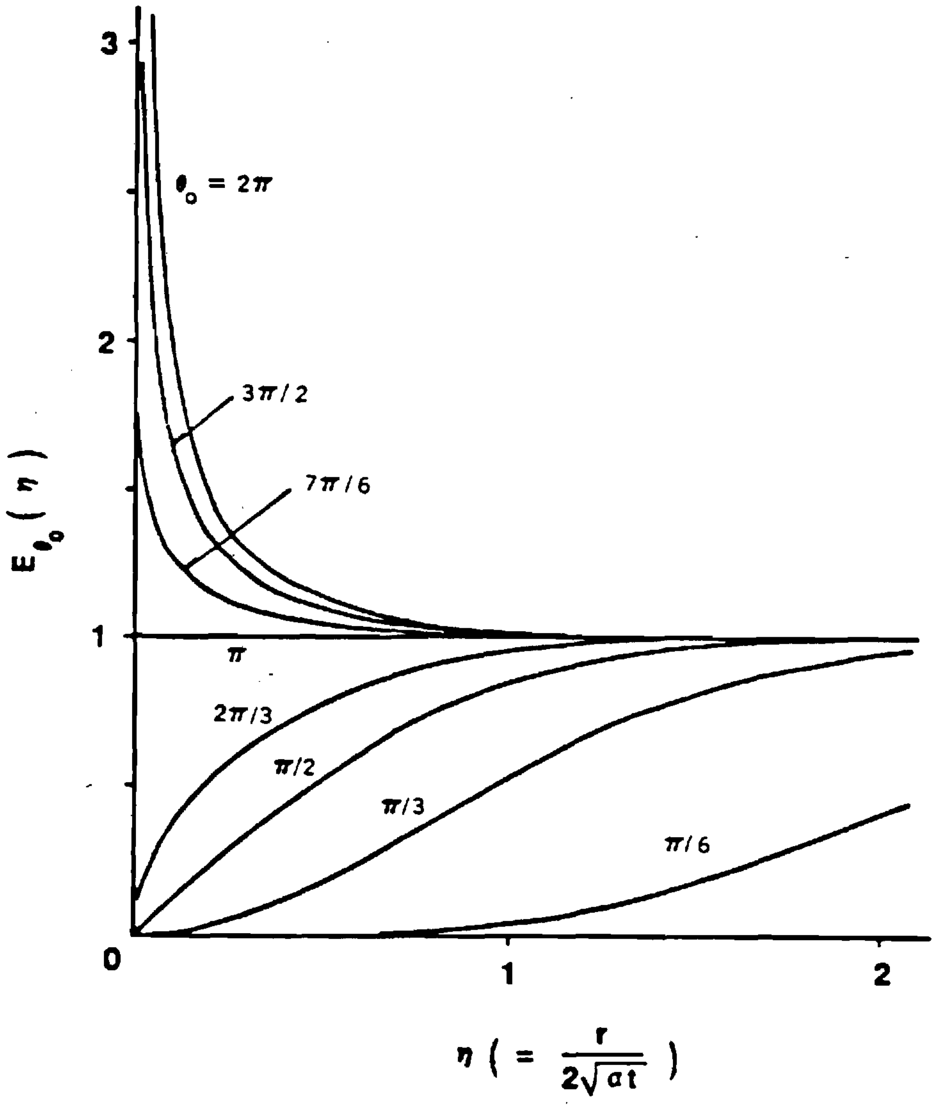

Figure VII-6. Graphical Representation of the E-function vs. $r/2\sqrt{\alpha t}$ (after Wei [12]).

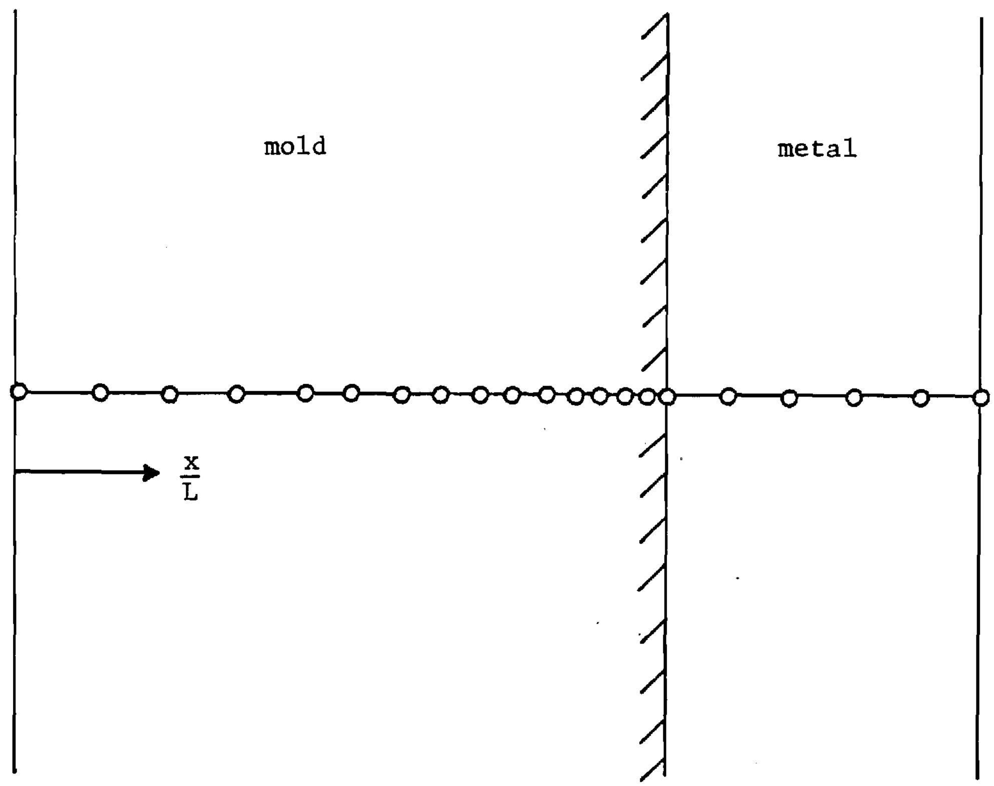

Figure VII-7. Mesh Used for the 1-D Case.

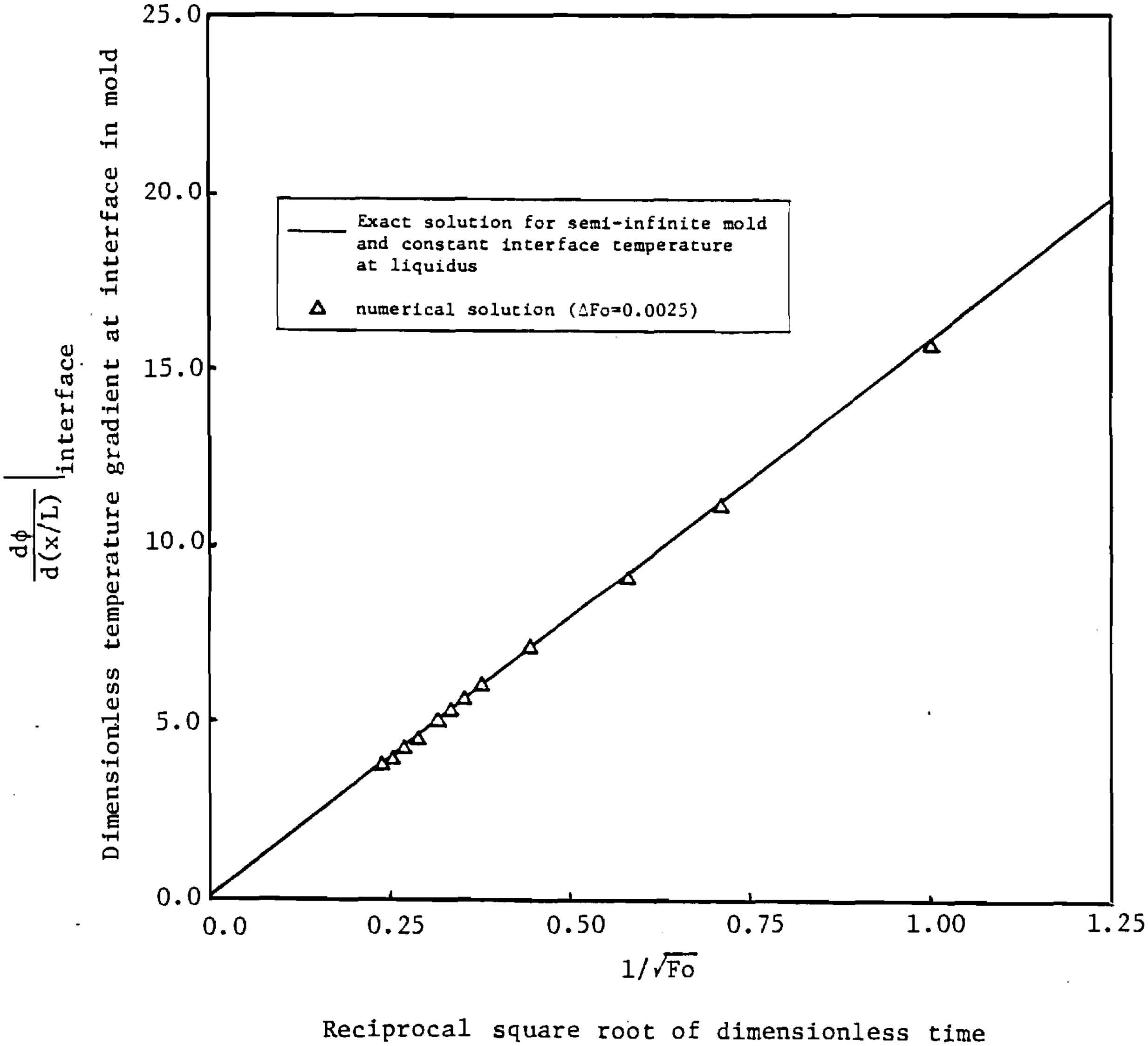

Figure VII-8. Numerically Calculated Dimensionless Temperature Gradients at the Interface as Compared to the Constant Interface Temperature Analytical Solution for the Aluminum/Silica Sand System ($Fo_m = \alpha_m Fo/\alpha$), Lumped Capacitance in Metal Only.

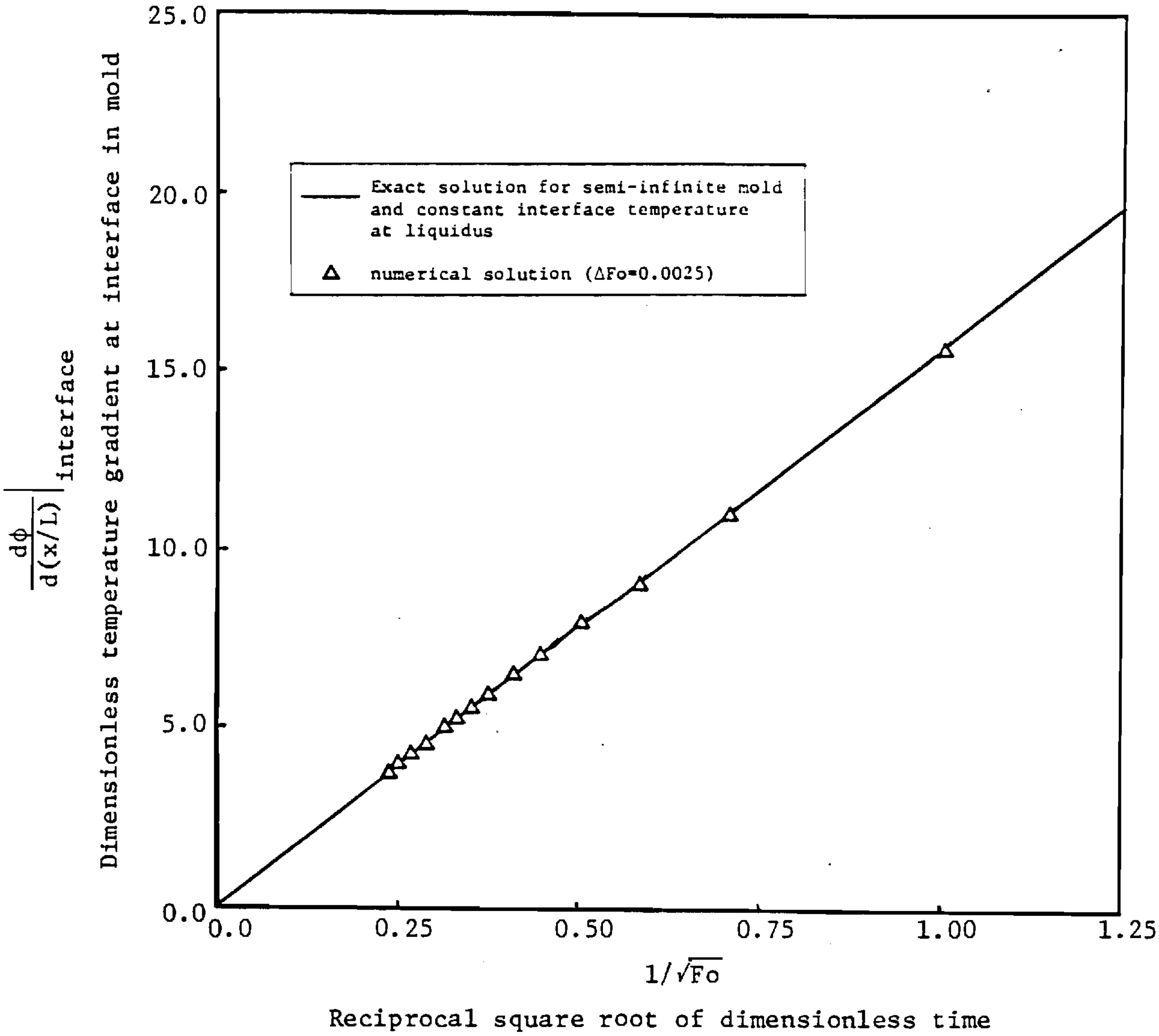

Figure VII-9. Numerically Calculated Dimensionless Temperature Gradients at the Interface as Compared to the Constant Interface Temperature Analytical Solution for the Aluminum/Silica Sand System ($Fo_m = \alpha_m Fo/\alpha$), Fully Lumped Capacitance Solution.

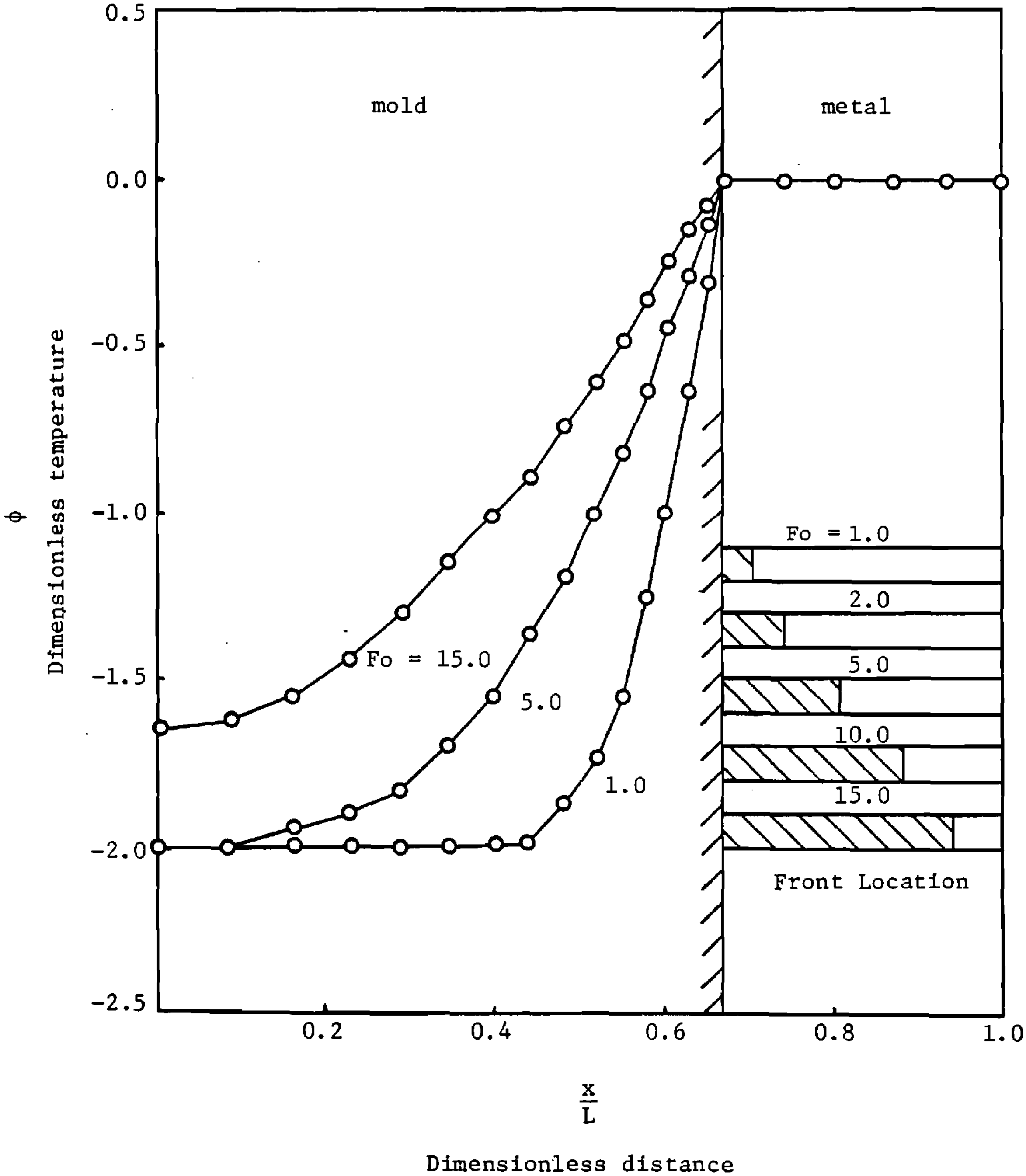

Figure VII-10. Dimensionless Temperature Profiles and Solidification Front Location at Various Values of Dimensionless Time, Lumped Capacitance in Metal Only.

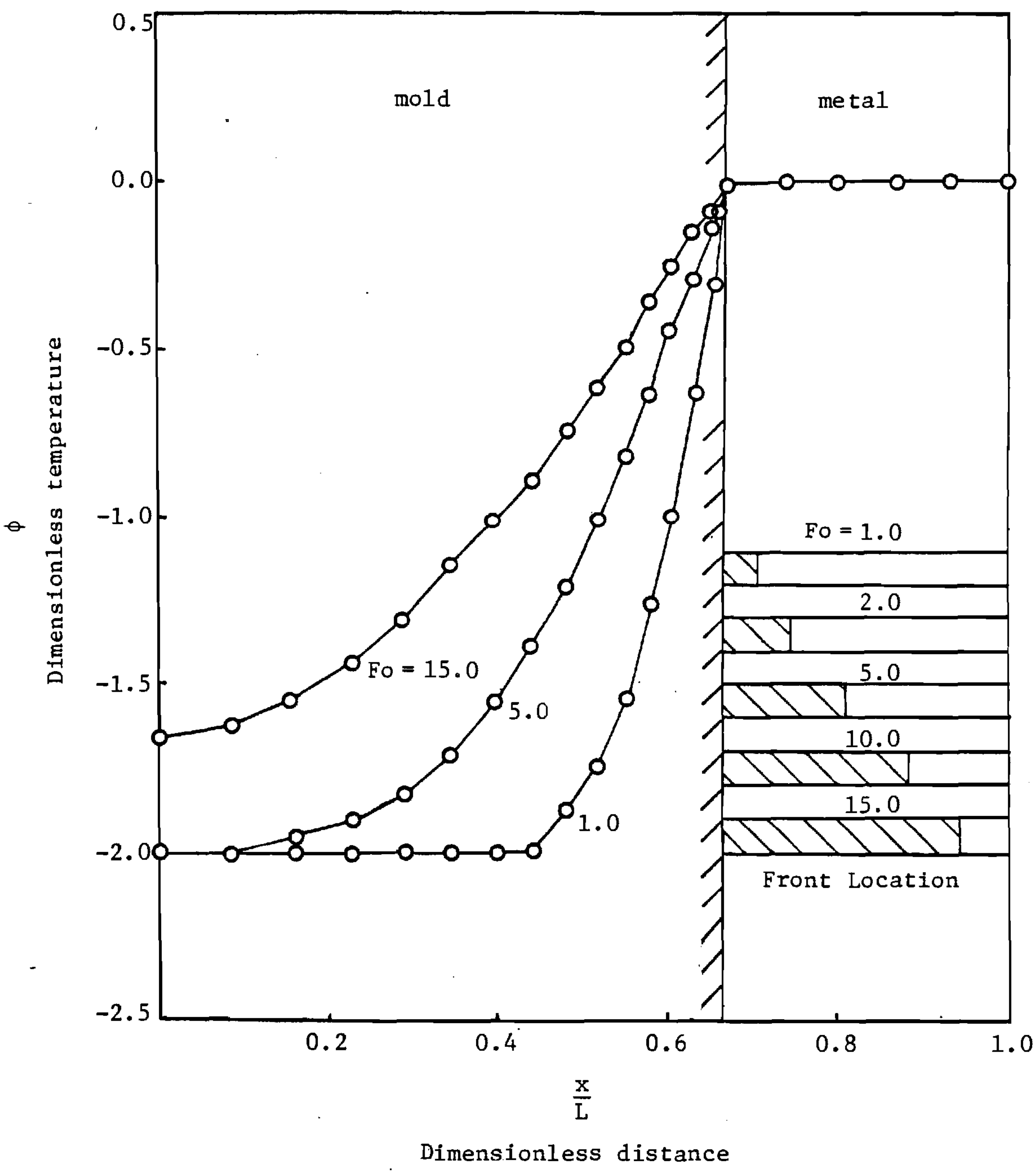

Figure VII-11. Dimensionless Temperature Profiles and Solidification Front Location at Various Values of Dimensionless Time, Fully Lumped Capacitance Solution.

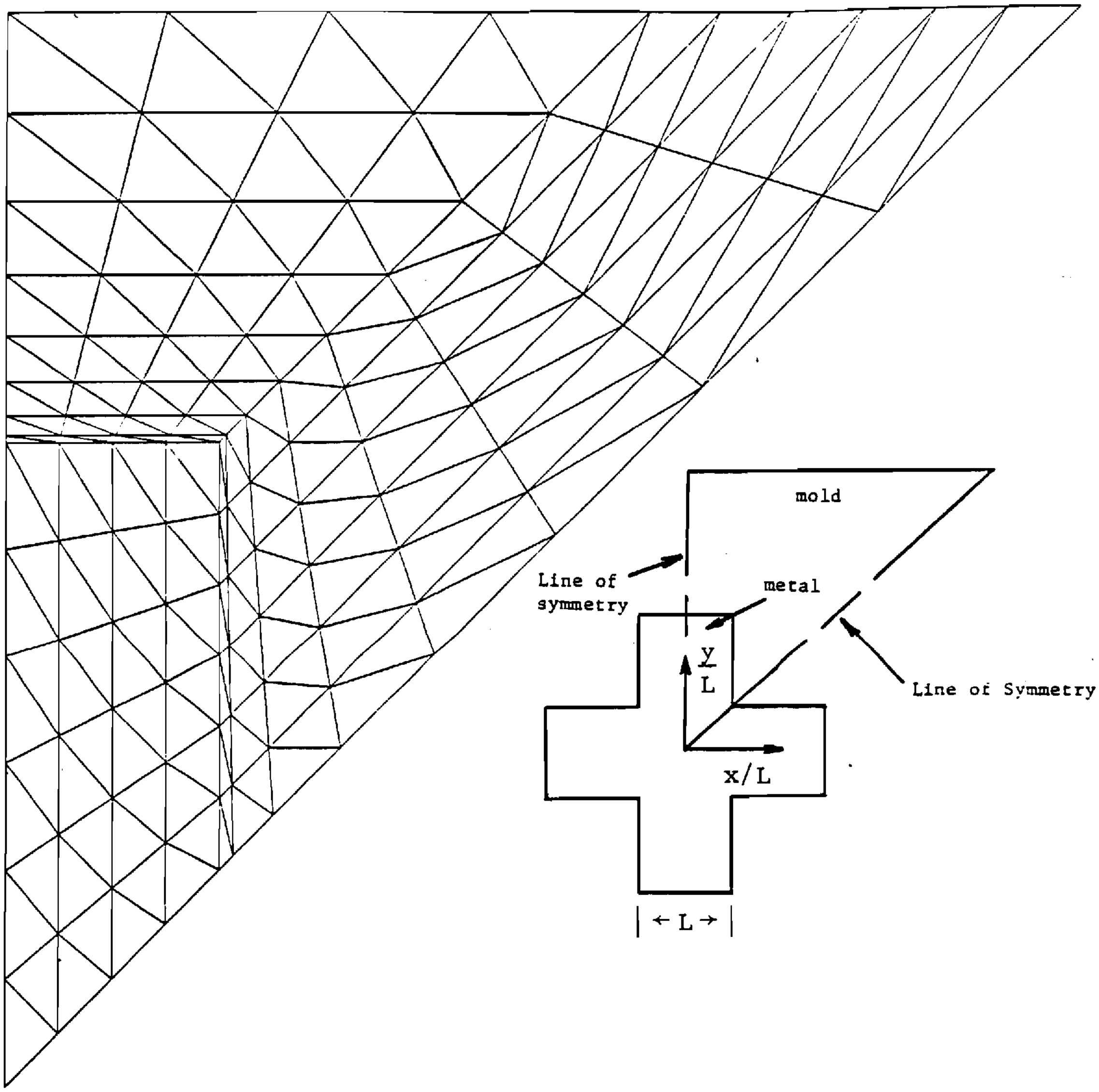

Figure VII-12. Mesh Used for the 2-D Cruciform Shape.

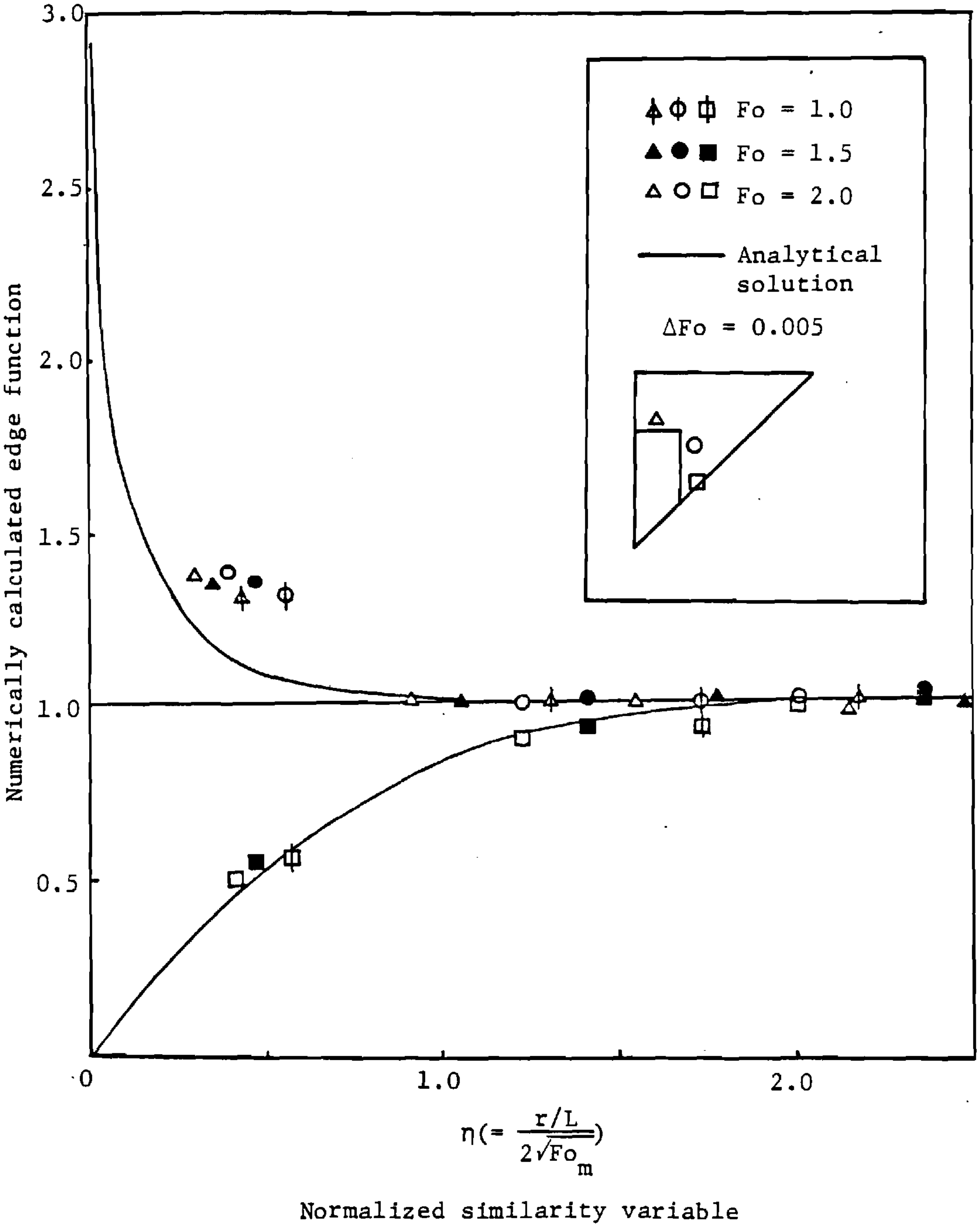

Figure VII-13. Numerically Calculated Values of the Edge Function $E(\theta_o) = \frac{\dot{Q}_{2d}}{\dot{Q}_{1d}}$ vs. $\eta(Fo_m = \alpha_m/\alpha Fo)$ Superimposed upon the Analytical Results of Wei [12].

APPENDICES

THE APPLICATION OF GEOMETRIC MODELING

TO METAL CASTING TECHNOLOGY

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Presented at the Symposium on

Solid Modeling by Computers: From Theory to Applications
Sponsored by the General Motors Research Laboratories

Warren, Michigan

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ABSTRACT

THE APPLICATION OF GEOMETRIC MODELING TO METAL CASTING TECHNOLOGY

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In the course of a large scale cooperative research effort between two major universities (Georgia Tech and the University of Michigan), several geometric modeling packages have been examined and utilized in the building of various casting/rigging configurations. Modelers scrutinized include TIPS, PADL-I, ROMULUS and CAT-I. The test castings involved have in turn included both simple laboratory type shapes, as well as industrial product type parts.

In general it has been seen that acceptable model forms can be built without major difficulty. In some cases, depending for example upon the availability of primitives, certain approximations are necessary; in other instances the number of allowable statements has dictated such approximations. Furthermore, it has been observed that work remains to be done on the questions of blending and the representation of other patternmaking needs such as parting line determination, although a surface type modeler DUCT has made extensive strides in the latter area.

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The other pressing need of metal casting engineers working in this area is seen to be applications type packages to connect geometric modelers with the general purpose of physical simulation schemes now available, as well as to those special purpose casting-oriented simulators under development.

The paper will describe the CADCAST team's experience with the present generation of modelers, together with comments upon their applications and their suitability towards fulfilling the needs of the industry in the near future.

INTRODUCTION

There are many potential roles for geometric modeling in the solidification processing area. These roles encompass both the larger aspects of foundry engineering such as the evolution of the layout of plant and equipment assemblies, as well as the more detailed features of the casting design and its relation to the rigging associated with the filling and feeding of the mold cavity.

The modeling of the casting/rigging system itself represents an important first step to the practicing foundry engineer. The foundryman is most often presented with a drawing or set of drawings which describes the geometry and topology of a casting on which he may be asked to make a quotation. Alternatively, he or she may be presented with a pattern or set of patterns at this juncture. In the future these forms of communication are liable to be replaced by sets of tapes, discs or by the direct linking of computer systems used by the foundryman and the designer respectively.

Having received the basic casting design information, the foundryman will add the rigging (gating and risering system). To undertake this, it appears most likely that a first approximation will be obtained by using one of the current available commercial rigging design software packages [Kotschi 1983; Ruddle and Suschill 1983; AFSoftware; Novacast Software; CRUSADER Software].

Some of the tasks that are generally required here are the provision of risers or feeder heads, which will provide a reservoir of molten metal sufficient to feed the shrinkage occurring during the cooling and freezing of the liquid metal in the casting proper. A further important task is integration of the total rigging system which promotes the quiescent entry of molten metal through a system of runners and gates into the mold cavity, as well as the feeding action through the feeders mentioned above. This first level software library [Kotschi 1983; Ruddle and Suschill 1983; AFSoftware; Novacast Software; CRUSADER Software] is not generally geometric modeling based, but using some simple scientifically-based axioms and certain well established items of codified empirical information provides useful working data for the foundry engineer. Such systems are normally run on personal/micro type computers. The riser design of such programs is based upon the modulus approach of [Chvorinov 1940] as modified by others, while the gating system portion follows the Bernoulli-based approach described in the literature (see, for example, [Flinn 1963]). Table I lists typical tasks such software can accomplish.

The system so designed would be the starting point for the geometric model building which would then take place. Here, the foundry methods engineer would literally build on the computer a three-dimensional model of the object to be cast together with its feeding and gating system.

Various castings have been modeled at Georgia Tech in the course of the Georgia Tech/University of Michigan CADCAST program employing a variety of modelers: TIPS-1, PADL-1, ROMULUS, CAT-1, DUCT (see reference listings). Though none of these programs is capable of modeling all casting geometries, the range of shapes that they can model is remarkable. The cost for licensing one of these modelers varies from about $250 to $100,000. The size of computer necessary to achieve reasonable response times with one of these modelers ranges from one with 128kbyte of memory and a 16-bit processor to one with 512kbyte of memory and a 32-bit processor. Sample shapes modeled by some of these programs are illustrated in Figures 1-3. On the average, the number of commands entered by the user in modeling one of these shapes is between 20 and 40.

Clearly many highly complex industrial castings will need an extremely large number of command statements. Consequently, modelers constrained in this fashion are of only limited use in describing commercial castings. Similarly, not all modelers are capable of blending surfaces; fewer still of locating parting lines, that is, the line associated with the plane dividing the upper (cope) and lower (drag) parts of the mold from each other. DUCT, however, has this feature [Welbourn]. Regarding complex surfaces showing double curvature, some modelers do permit insertion of user defined elements [TIPS-1 1978]. However, since this subject will be the topic of a whole session at this conference, let it be sufficient to add that such capabilities are extremely desirable in any modeler to be used in connection with castings applications.

MODELING REAL CASTING ASSEMBLIES

One of the castings modeled, that depicted in Figure 3, was in fact poured in a commercial foundry [Berry 1982]. Although the casting itself is simple in configuration, being essentially a thick-walled hollow cylinder, the rigging is of special interest because of its complex design.

The casting, a machinability test log [Berry 1982], is poured and fed through a complicated series of shapes, each having its own particular role. The system concerned contains features which perform three functions:

(a) a gating system which conducts clean molten metal into the casting cavity in an even manner;

(b) a reaction chamber which permits the treatment and inoculation of the molten metal;

(c) a feeding system which acts as a reservoir of molten metal capable of transmitting liquid to the cooling casting proper.

Figure 3 shows the assembly concerned split along a vertical plane which passes through the down sprue to which liquid metal is supplied from the pouring ladle. On either side of the down sprue is situated the reaction chambers. It is in these chambers, the dimensions of which are strictly governed by practice, that the desulfurization and inoculation of the molten metal (cast iron in this instance) takes place. Figure 4 shows a partial model of this portion of the rigging.

The horizontal passages leading out of each chamber contain slag traps, so that any ingested slag or undesirable reaction products are not conducted into the casting. The portion of the rigging which runs parallel to the axis of the cylindrical casting will not commence to fill until the slag trap has been traversed, thus promoting the trapping motion. The actual runner bar, which is parallel to the above axis, is also designed in such a way as to promote the even entry of metal into the ingates. This is accomplished by controlling both the runner bar and ingate cross-sectional areas.

Clearly the possession of a geometric modeling facility would permit the foundry engineer to check out in detail many aspects of the above system. Simple commands allow him or her to section on the (horizontal) parting plane (Figure 5), in the (vertical) plane passing through the sprue and the reaction chamber (Figure 6) or in a parallel vertical plane which passes through the ingate-riser-casting region (Figure 7).

Having checked out intrinsic features of such components of the rigging, the engineer concerned can determine whether there are design aspects of the casting which might be adjusted to the advantage of its castability. It is at this point where the exciting possibility of the foundryman conversing with casting designer whilst both are viewing the modeler emerges. Such rapid and obviously effective means of communication would clearly promote better understanding between the designer and the supplier.

A second important commercial aspect of the potential of the geometric modeling of castings hinges upon casting yield. This quantity essentially measures the efficiency of a particular rigging system. Indeed, it may well constitute one of the first points of contact between the foundry and the designer. To determine the selling price of a casting the foundry methods engineer must know just how much metal has to be melted and poured to produce one sound and properly dimensioned casting. Use of the modeler permits a tremendous saving here. The routine adopted might be:

(1) INPUT THE COMMAND STATEMENTS TO MODEL CASTING ONLY (thick-walled cylinder in this case).

(2) USE PRE-PROCESSOR ROUTINES AND VOLUME CALCULATION ROUTINE TO DETERMINE CASTING VOLUME.

(3) INPUT THE COMMAND STATEMENTS TO MODEL THE RIGGING ONLY.

(4) USE PRE-PROCESSOR ROUTINES AND VOLUME CALCULATION ROUTINE TO DETERMINE RIGGING VOLUME.

(5) UTILIZE THE RESULTS OF (2) AND (4) TO COMPUTE YIELD.

As a final check of the suitability of a given rigging design, the foundry methods engineer would link the geometric modeler with a physical simulation system. This link would permit the person concerned to literally pour and freeze the model on the computer. Alternatively, with a more complex design of casting, one might only wish to check out certain features of the system concerned. The linking together of the modeler and the simulator is one of the present areas of concern of the Georgia Tech/University of Michigan team and, although out of the scope of the present paper, has been discussed by them elsewhere [Boulet and Dalton 1982]. At present this is an area where concentration of research effort is critically needed.*

*For a more extended discussion of this important topic, the reader is referred to the progress reports of the CADCAST project underway at the Georgia Institute of Technology and the University of Michigan.

CONCLUSIONS

Basic concerns of the casting engineer include the yield, soundness and material properties of his product. The ability to predict these qualities is a prerequisite for effective design of casting and rigging. With the advent of relatively inexpensive graphical display devices, interactive programming techniques, and advanced numerical methods for the solution of nonlinear partial differential equations, it is now possible to increase significantly the foundry methods engineer's ability to predict all aspects of a casting before pouring. The potential increase in productivity associated with implementation of geometric modeler based computer-aided design techniques will not be fully realized for some time to come, but the day when one can "pour it on the computer" is approaching. Although the present generation of geometric modelers does not contain all of the routines that are of interest to the pattern maker or the foundry methods engineer, good approximations of many real castings can be readily built. Similarly, much work needs to be done on linking together geometric modelers and casting solidification simulation packages.

The present paper has described some of the tasks of the foundry methods engineer and has shown these tasks can be more effectively accomplished utilizing geometric modeling and solidification simulation techniques.

The potential for increased productivity of the individual methods engineer, the enhancement of the system of communication between the methods engineer and the casting designer and finally the more accurate estimation of casting costs are all highly encouraging.

ACKNOWLEDGEMENTS

The work described in the paper forms part of the CADCAST program underway at the Georgia Institute of Technology and at the University of Michigan (Program Director at the University of Michigan is Prof. Robert D. Pehlke). The program is supported by the National Science Foundation (Program Manager, Dr. William Spurgeon) and by industry. At Georgia Tech further support from the State of Georgia is gratefully acknowledged.

The contributions of Mr. F. Balboni, formerly of Georgia Tech, now with IBM Corporation in the area of model building, are acknowledged. The team also acknowledges help received from the organizations supplying the TIPS-1, PADL-1 and ROMULUS models illustrated in the paper.

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CatGuide, The Users Manual. CatSoft 1.1. May be obtained through the Catronix Corporation, 120 Ralph McGill Boulevard, N.E., Suite 800, Atlanta, GA 30308, ATTN: Dr. Jo Ellen Bradham.

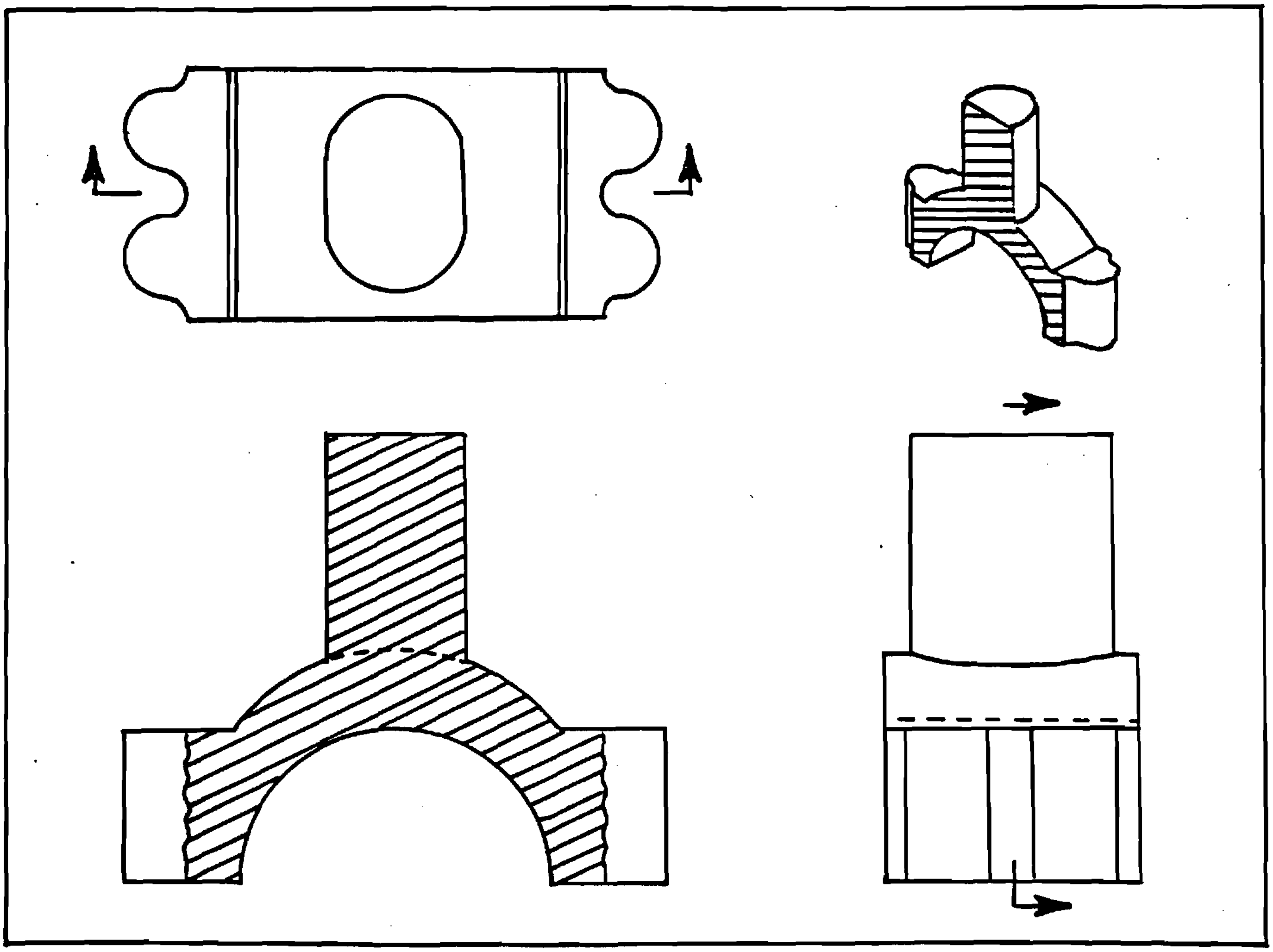

Figure 1. Bearing Cap Modeled with PADL-1.

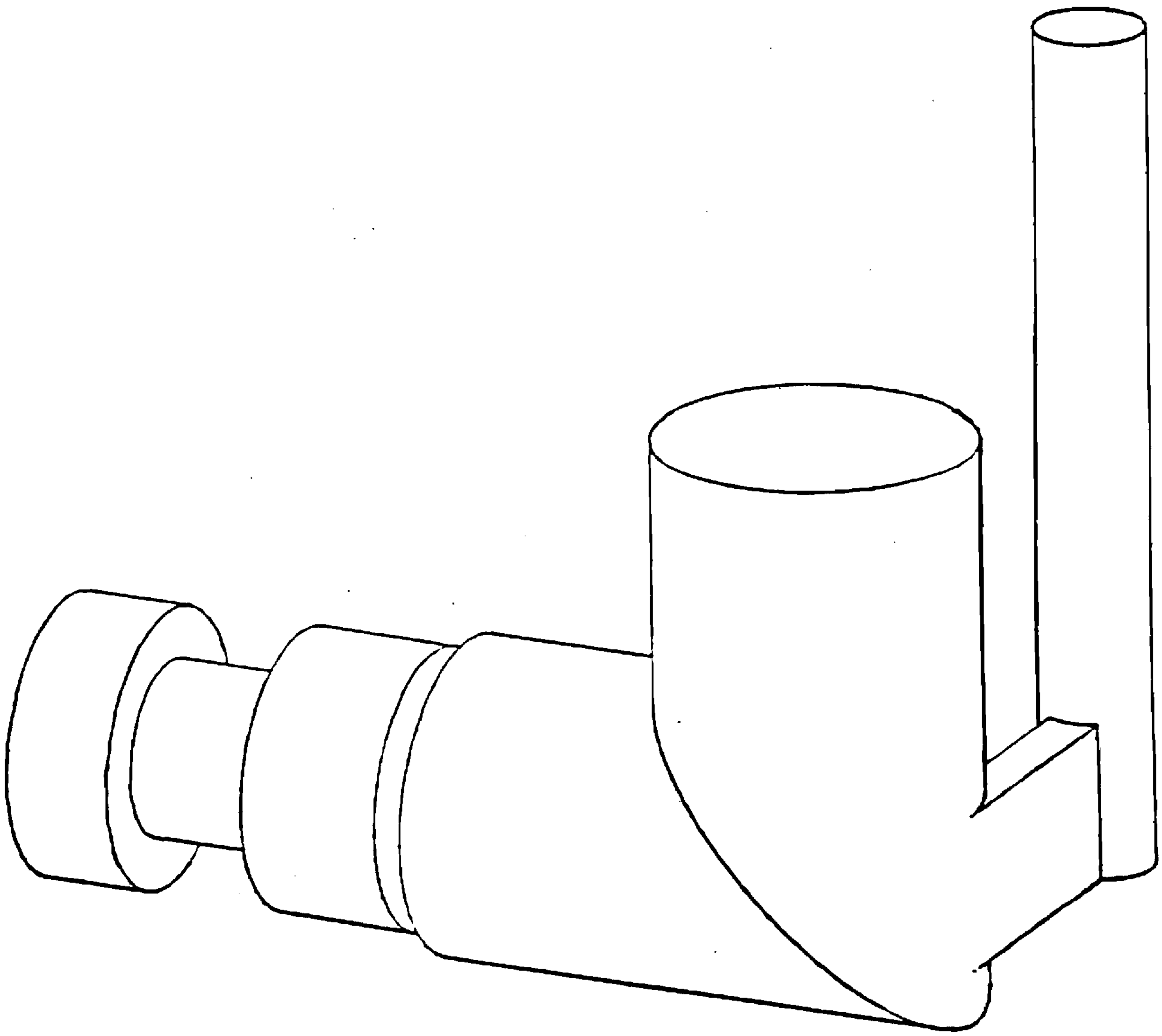

Figure 2. Test Casting Modeled with ROMULUS.

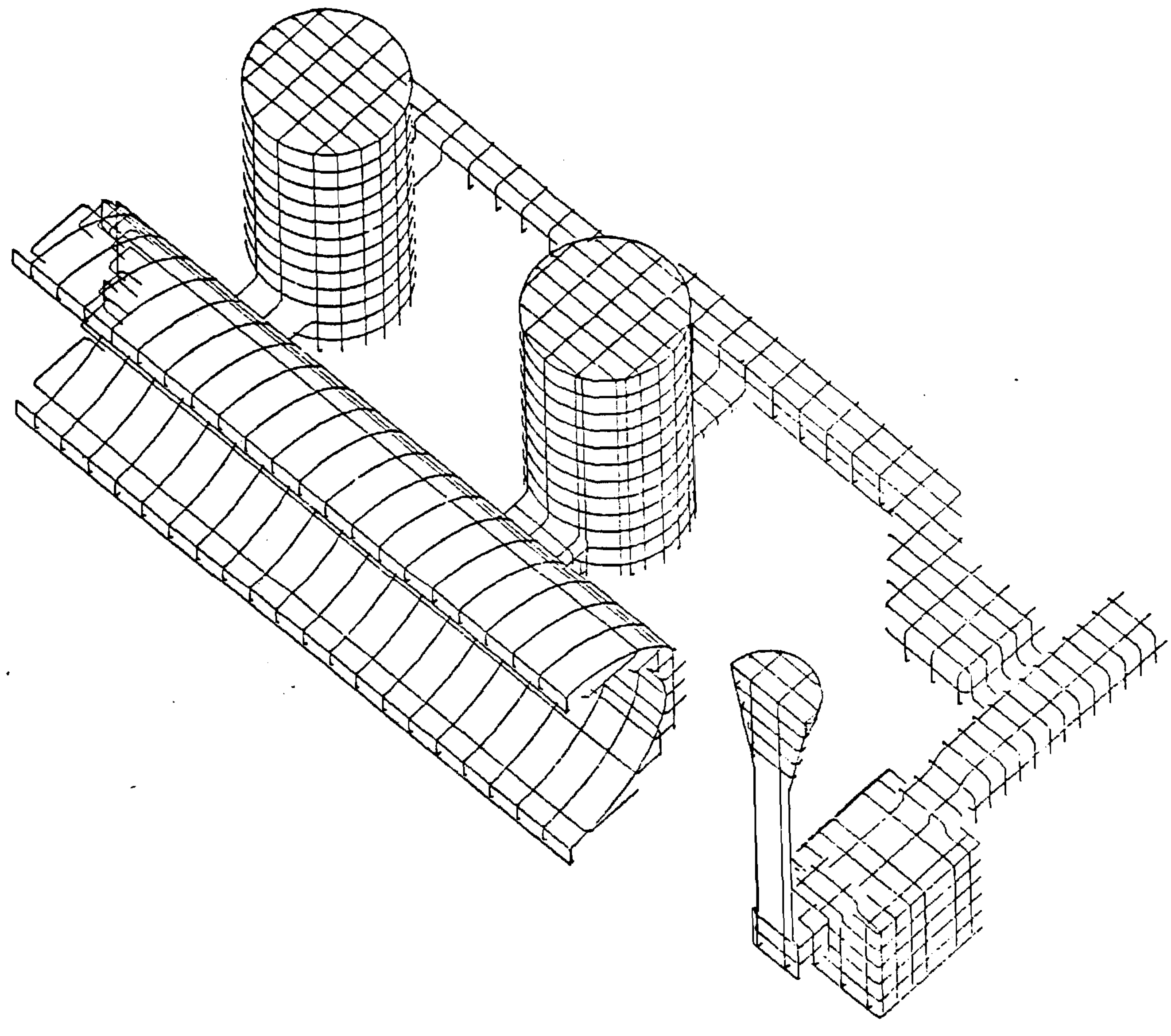

Figure 3. Machinability Test Piece with Commercial Foundry Rigging, Modeled Using TIPS-1.

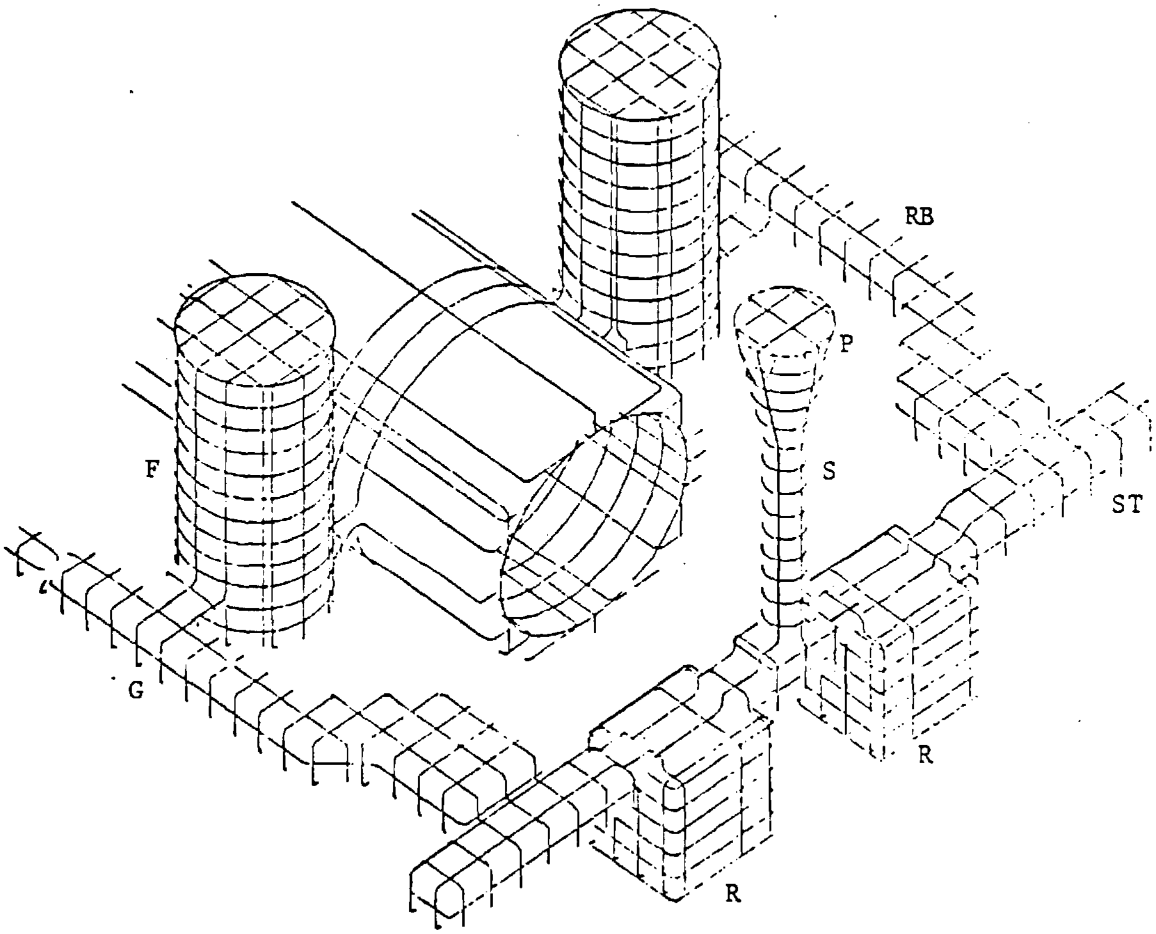

Figure 4. Showing Portion of the Rigging System from Figure 3, Showing Reaction Chambers and Slag Traps. (TIPS-1 Model)

F Feeder or Riser

G Ingate location

P Pouring basin

R Reaction chamber

RB Runner bar

S Sprue

ST Slag trap

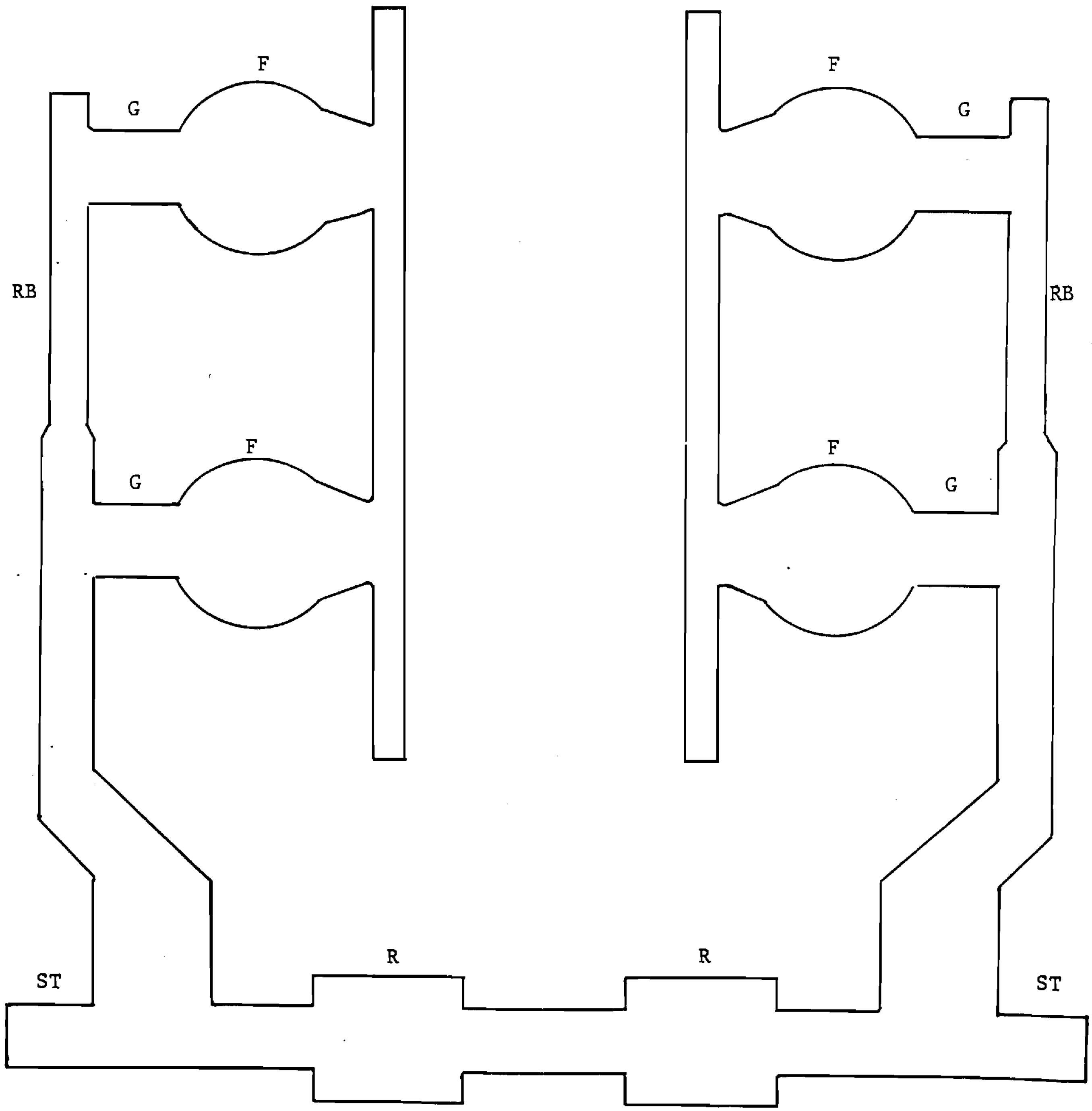

Figure 5. Section Made on Parting Plane. (TIPS-1 Model, Refer to Figure 3.)

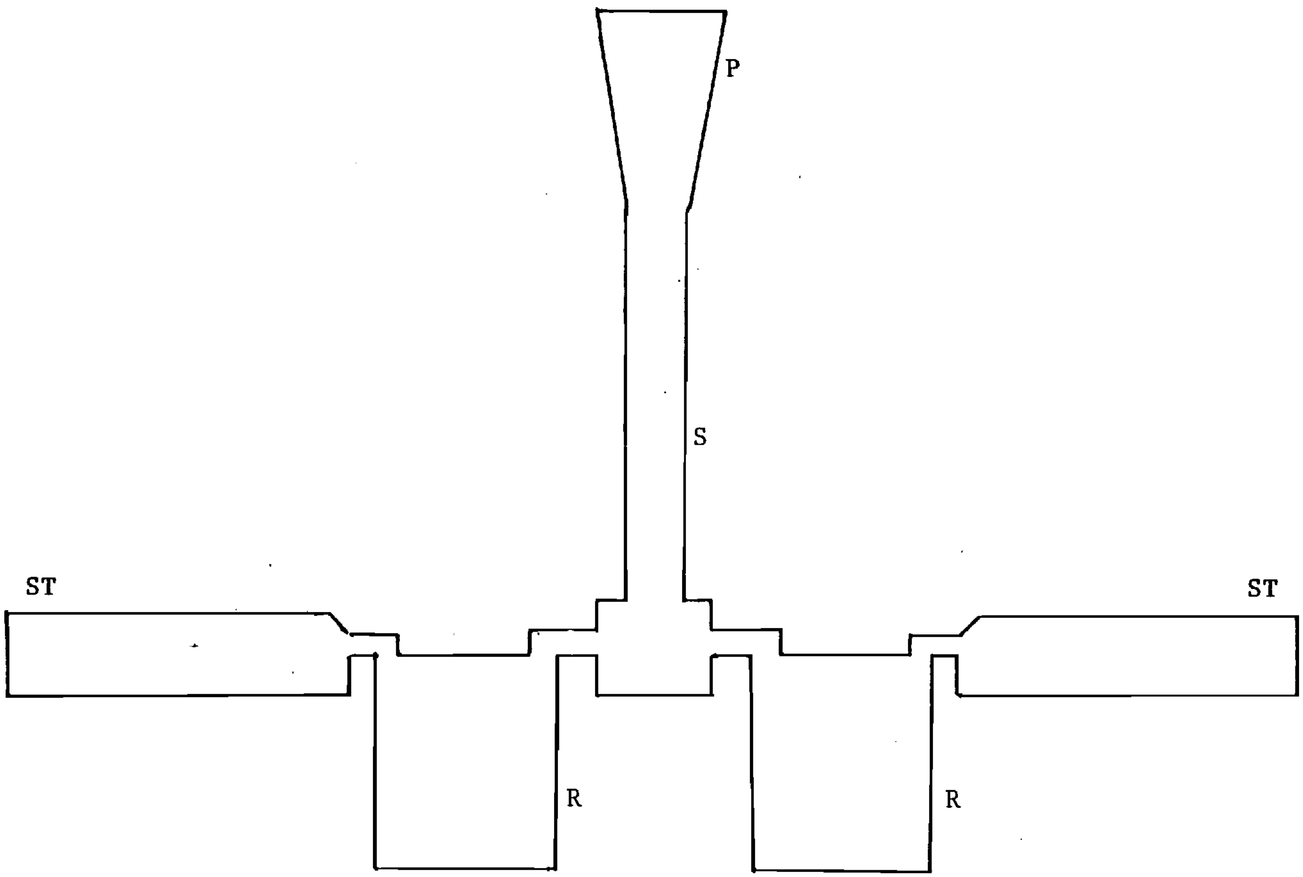

Figure 6. Section Made through Sprue, Reaction Chambers and Traps.
(TIPS-1 Model, Refer to Figure 3.)

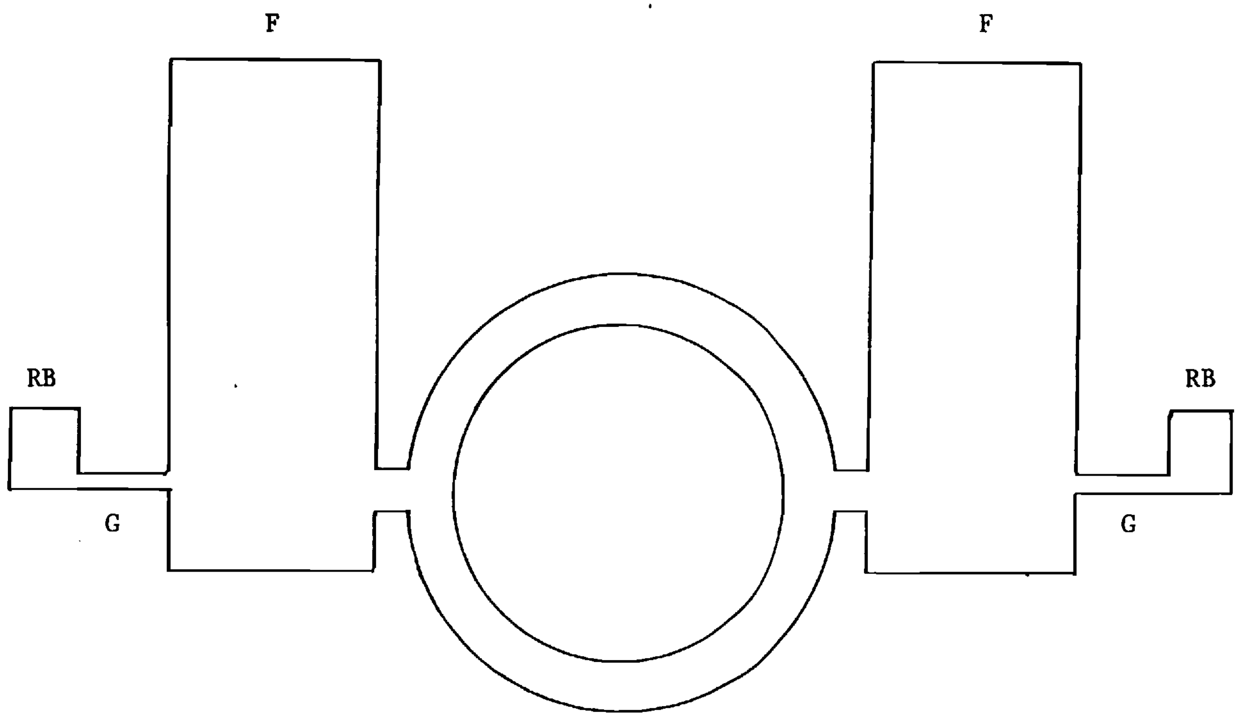

Figure 7. Section Made through Risers and Ingates.
(TIPS-1 Model, Refer to Figure 3.)

ELEMENTS OF RIGGING DESIGN

- Dimensioning of Risers
- Positioning of Risers

- Placement of Chills

- Padding or Modification of Casting Design Features

- Design of Gating System

Table 1. Typical Foundry Engineering Tasks.

GLOSSARY

Gating System: That part of a casting which conducts the molten metal from the entrance into the mold through to the mold cavity associated with the shape of the cast. In turn, the elements consist of the pouring basin, the down sprue, the various runner bar elements, and finally the gates which admit the molten metal to the actual mold cavity for the shape to be cast. Specialized elements that might be incorporated into such a system are a sprue well and slag traps which are designed to promote cleanness of the metal stream and in some cases a reaction chamber for in situ treatment of the molten metal, as in the Inmold™ process. See Figure 4.

Inoculation: A term used to describe a treatment given to molten gray or ductile cast irons to refine the eutectic graphite flake size. This is accomplished by adding a powdered material to either the pouring ladle or the metal stream as it enters the mold. Alternatively, the powdered inoculant may be placed within a reaction chamber within the gating system. Such chambers may also contain nodularizing agents to change the morphology of the graphite from a flake to a spheroidal variety.

Parting Line: In order to separate the sand mold from the pattern in a convenient manner, the pattern maker designates a parting line (or split line) which will separate the upper part of the mold (the cope) from the lower part (the drag). Although in simple cases the line concerned will remain on one plane (the parting plane), in more complex shapes a series of planes and even portions of curved surfaces may be involved. Location of this plane is currently regarded as one of the important aspects of the art of pattern making.

Risers: Risers, often described as riser heads or feeder heads, are also an important part of the casting rigging. They are essentially meant to act as reservoirs of molten metal which will feed the liquid metal within the mold cavity for the part to be cast as it cools and subsequently solidifies. They are generally located adjacent or in very close proximity to the mold cavity.

A THEORETICAL STUDY OF THE USE OF HEAT PIPES IN METAL CASTING

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ABSTRACT

A theoretical study has been performed of the effects of using a heat pipe alone and a heat pipe in conjunction with a convential chill in the casting of metals. The main goal of the project was to determine the feasibility of using heat pipes to control rates and directions of solidification, temperature gradients during solidification and cooling, and micro-structure. A transient model for the heat pipe and two-dimensional governing thermal equations for the casting were developed. The resulting equations were transformed to finite-difference form and were solved using alternating implicit techniques. A small 60-40 lead-tin alloy casting was studied using heat pipe only, chill only, and heat pipe/chill combination. It was found that the use of a heat pipe greatly alters the direction and velocity of the freezing front and very importantly the temperature gradients at the freezing front.
KEYWORDS: Heat Pipe, Casting, Transient Analysis

INTRODUCTION

The concept of using heat pipes for control of heat flow during the casting of both metals and non-metals has previously been studied by several researchers. However, to the authors' knowledge, no previous attempt has been made to incorporate a transient heat pipe model with a transient two-dimensional casting model where the resulting governing equations were solved by finite difference techniques to obtain detailed time dependent solutions. Wells (1) gives a detailed account of the theoretical work and results which are only briefly summarized herein.

Bahadori (2) analyzed the casting of aluminum, cast iron and tin while applying variable conductance heat pipes. He demonstrated that casting rates were affected by the heat pipes. Mascaretti and Medana (3) discuss the use of heat pipes to cool parts of a casting die and MODEM (4) indicates that heat pipes may be used in plastic injection molding. Feldman (5) examined the use of heat pipes in plastic mold cooling systems. The pipes were used to cool the mass of the mold rather than to cool the part.

It has long been known that the history of solidification of a metal is directly related to final internal structure and mechanical properties of cast parts. St. John, Wu and Berry (6) experimentally studied the effects of directionally solidifying an aluminum alloy. It was determined that temperature gradients and rates of solidification controlled microstructure and ductility of the alloy. Berry (7) concluded that ductility might be controlled in the foundary by preprogramming solidification using computer simulation.

Colwell and associates (8,9,10) have for many years been involved in modeling and measuring both steady state and transient operations of heat pipes. The work has included studies of the affects of capillary structure drying and rewetting, startup from the thermodynamic supercritical state, and vapor velocity distribution variations. Finite difference and finite element numerical techniques have been used to study the governing equations and associated boundary and initial conditions.

THEORY

The casting was assumed to be in the form of a semi-infinite slab with a chill at the bottom and heat pipes on opposite sides as shown in Figure 1. Only one heat pipe, half of the chill and half of the casting are shown in the figure since a plane of symmetry exists half way between the two heat pipes. Both heat pipes and the chill are liquid cooled outside the mold cavity.

Figure 2 shows a heat pipe with cooling jacket and heated through one surface encompassed by a control volume boundary. A highly simplified transient equation for the control volume is

$$Q_{in} - Q_{out} = C_{HP} \frac{\partial T_p}{\partial t} \qquad [1]$$

where

Q_{in} = energy entering heat pipe from casting at an instant of time

Paper presented at Fifth International Heat Pipe Conference, Tokyo, Japan, May 1984.

Q_{out} = energy leaving heat pipe and entering coolant at an instant of time

C_{HP} = heat capacity of heat pipe including shell, capillary structure and working fluid

T_p = mean bulk temperature at an instant of time

t = time

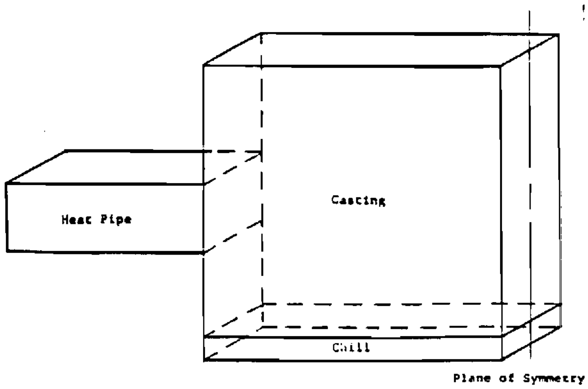

Figure. 1. Schematic of Heat Pipe and Casting Geometry

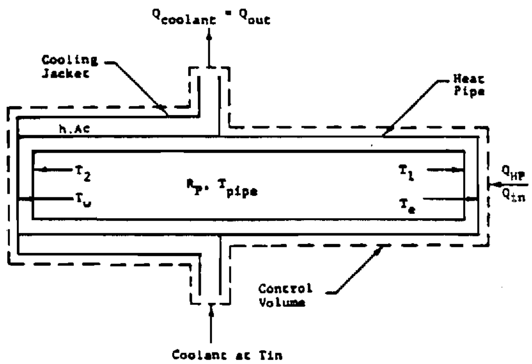

Figure 2. Control Volume Used for Resistance Capacitance Model for the Heat Pipe.

assuming that at an instant of time

$$T_1 \cong T_p \cong T_2$$

i.e., at an instant of time temperature gradients in the vapor region are small,

$$Q_{HP} = \frac{hA_c R_r}{hA_c R_c + R_r} (T_e - T_{in}) + \frac{C_{HP} R_r (hA_c R_c + 1)}{(hA_c R_c + R_r)} \frac{dT_e}{dt} \qquad [2]$$

where

Q_{HP} = energy leaving casting and entering heat pipe at an instant of time

h = convective cooling coefficient on outside of heat pipe in cooling jacket

A_c = outside area of heat pipe in cooling jacket

$R_r = \frac{R_c}{R_c + R_e}$ = resistance ratio

R_c = thermal resistance between vapor and outside of heat pipe at condenser end

R_e = thermal resistance between vapor and outside of heat pipe at evaporator end

T_e = outside wall temperature at evaporator

T_{in} = temperature of cooling fluid

The thermal resistances between vapor and outside of heat pipe shell must account for resistance of capillary structure, liquid layers, and the shell itself. Williams (10) gives an equation for determining the effective thermal conductivity of a single layer of screen filled with a liquid.

$$\frac{k_{eff}}{k_\ell} = \frac{1}{[\frac{r_c}{r_{ws}} + 1]\,[2\ \frac{k_\ell}{k_s} + \frac{r_c}{r_{ws}} - 1]} + \frac{2}{[\frac{r_c}{r_{ws}} + 1][\frac{k_\ell}{k_s}\frac{r_{ws}}{r_c} + 1]} + \frac{1}{[\frac{r_{ws}}{r_c} + 1]^2} \qquad [3]$$

where

r_c = wick pore radius

r_{ws} = wick solid radius (1/2 wire diameter for mesh screen)

k_s = wick solid thermal conductivity

k_ℓ = liquid thermal conductivity

The casting was broken up into a number of elements in the vertical and horizontal directions as shown in Figures 3, 4 and 5. Figure 3 shows the region where the heat pipe and casting meet and Figure 4 shows the region where the chill and the convective boundary meet. Figure 5 shows an interior mode in the casting. A heat balance at mode p in Figure 5 gives

$$Q_N + Q_S + Q_E + Q_W = mC\,\frac{\partial T}{\partial t} \qquad [4]$$

where

$$Q_N = \left(k\,\frac{\partial T}{\partial y}\right)_n A_n \qquad [5]$$

$$Q_S = \left(k\,\frac{\partial T}{\partial y}\right)_s A_s \qquad [6]$$

$$Q_E = \left(k\,\frac{\partial T}{\partial x}\right)_e A_e \qquad [7]$$

$$Q_W = \left(k\,\frac{\partial T}{\partial x}\right)_w A_w \qquad [8]$$

m = mass in elemental volume at P
C = heat capacity of material at p.

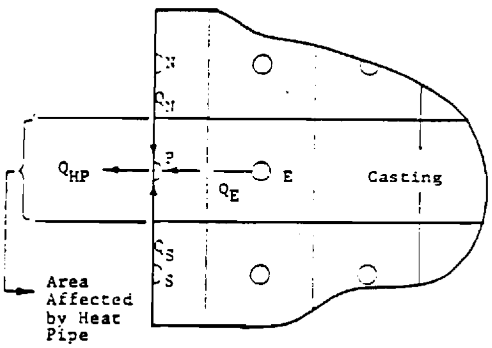

Figure 3. Energy Balance on a Typical Heat Pipe Boundary Node

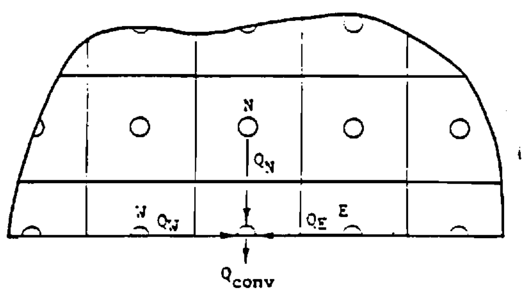

Figure 4. Energy Balance on a Typical Convective Boundary Node on the Lower Chill Surface.

The heat balance can be approximated by

$$\alpha_n(T_n - T_p) + \alpha_s(T_s - T_p) + \alpha_e(T_E - T_p) \qquad [9]$$

$$+ \alpha_w(T_w - T_p) = \frac{\Delta x^2}{\Delta t}\,(T_p^{\,1} - T_p^{\,0})$$

where

α = $\frac{\;}{\rho C}$ = thermal diffusivity

Δx = Δy

$T_p^{\,1}$ = temperature at p after Δt

$T_p^{\,0}$ = initial temperature at p

At the boundary between the heat pipe and the casting the balance becomes

$$Q_N + Q_S + Q_E + Q_{HP} = mC\,\frac{\partial T}{\partial t} \qquad [10]$$

and at the boundary between the chill and the coolant the balance becomes

$$Q_N + Q_E + Q_w + Q_{conv} = mC\,\frac{\partial T}{\partial t} \qquad [11]$$

Q_{HP} in equation [10] is given by equation [2] and Q_{conv} in equation [11] is determined by a film coefficient and the coolant temperture.

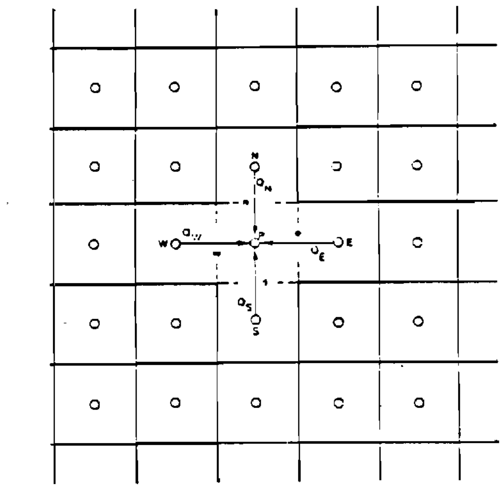

Figure 5. Grid and Labeling Method Used to Derive General Finite Difference Equations.

Equations [9], [10] and [11] when applied throughout the casting yield a set of algebraic equations which may be solved to give transient temperature distributions within the casting and transient conditions within the heat pipe. An implicit alternating direction method [9] has been used to solve these equations. The implicit alternating direction method employs two sets of difference equations which are used, in turn, over successive time steps, each of duration $\Delta t/2$. During the first half-time step the difference equations are implicit in the x direction and explicit in the y direction. While in the second half time step the difference equations are implicit in the y direction and explicit in the x direction.

In order to solve the difference equations described above the thermal diffusivity must be known at every point in the casting at each instant of time during solidification and cooling. Thus, thermal conductivity, specific heat, and density must be evaluated at every point at every instant of time taking into account large temperature changes and changes of phase. The procedure used in this study was to utilize an equilibrium diagram for the alloy being cast along with thermal property data for the pure metals in solid and liquid form. Wells [1] describes the procedure in detail

SYSTEM STUDIED

The casting of a 60-40 lead-tin alloy using copper heat pipes and a copper chill was studied. The mold cavity was 5 cm in width, 9 cm in height, and 2.5 cm in depth. The heat pipes were square in cross-section having sides of 2.5 cm. Evaporator area was 6.25 cm^2, condenser area was 50 cm^2 and the adiabatic length was 5 cm.

Heat pipe wall thickness was taken to be 0.1588 cm and two layers of 100 mesh copper screen were used as the capillary structure. A small fluid gap (0.00305 cm) was assumed between the layers of mesh. The heat pipe working fluid was water and both the heat pipe and the chill were water cooled.

The numerical model of the solidification process included temperature dependent properties in the alloy and artificially incorporated the effects of latent heat release during solidification. The properties of the heat pipe shell, wick and working fluid were taken to be constant. The heat pipe, metal being cast, and chill were taken to be isothermal at 574K which corresponds to the appropriate superheat for the alloy . Solidification occurs at about 456K. For the data presented herein, an intimate contact was assumed at heat pipe/casting and chill/casting interfaces. Work is currently underway to incorporate the effects of gaps at these interfaces into the model.

RESULTS

The numerical model was used to study three different casting/heat sink systems. These systems were casting/heat pipe only, casting/chill only and casting/heat pipe and chill together. The heat pipe and chill are cooled by convective means, the heat pipe being cooled by a coolant passing through a cooling jacket outside the condenser end, and the chill being cooled either by a cooling jacket or a jet spray impinging on its lower surface. Values of 1 and 5 $\frac{W}{cm^2K}$ were used for convective coefficients under various cases. In a particular computation the same value for convective coefficient was used in the heat pipe cooling jacket and on the outside of the chill.

Figure 6 shows the amount solidified as a function of time using heat pipe and chill. Heat pipe, molten metal, and chill were initially at 574K and at time zero water at 313K was introduced at the outer chill surface. About three minutes was required for complete solidification with chill only, about 3.3 minutes with heat pipe only, and about 2 minutes with heat pipe and chill. It should be noted that coolant was not introduced to the heat pipe until the freezing front reached the heat pipe location in the case where heat pipe and chill were used. Figure 7 shows progress of the freezing front with time. The heat pipe is started after about 0.6 minutes.

Figure 8 shows position of the freezing front on several planes for chill only and chill and heat pipe. Figure 9 gives locations of the planes.

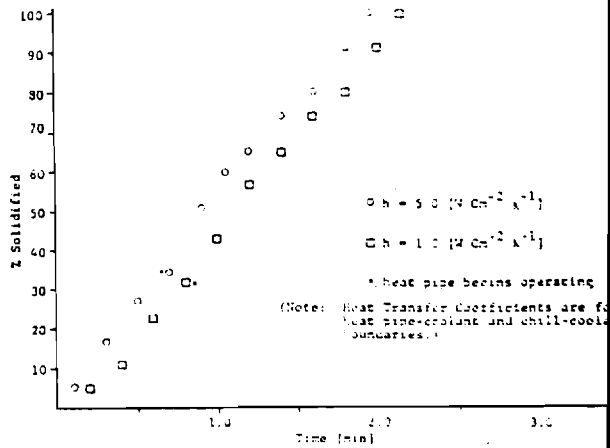

Figure 6. Percent of Total Volume Solidified as a Function of Time Using the Heat Pipe and Chill in Combination.

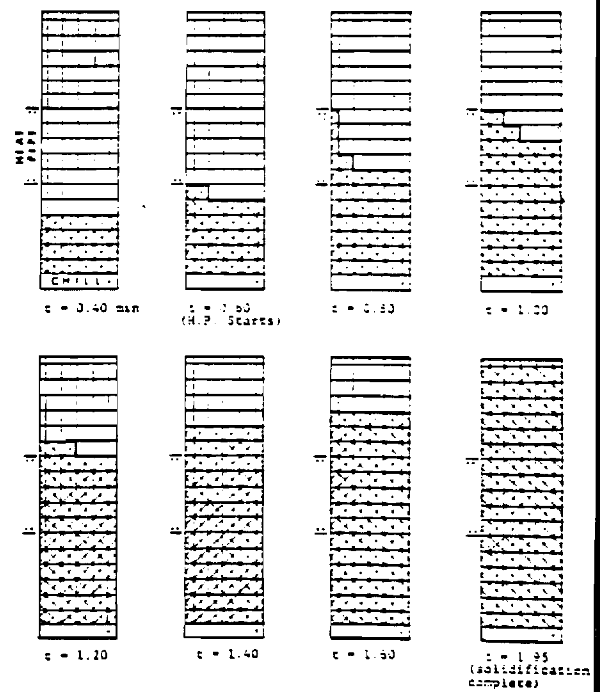

Figure 7. Influence of Using the Pipe and Chill in Combination on the Solidification Pattern.

As seen in Figure 10, the heat pipe dramatically affects freezing velocities. Figure 11 shows temperature gradients at the solid-liquid interface as a function of time. The two dimensional and discrete native of the numerical model made it necessary to establish a consistent method for calculating the temperature gradient at the interface. In all cases the temperature gradients were calculated in the y-direction only by using the temperature at the solidified interface node and the temperature of the node directly above it, that is, a gradient from the solid material to the liquid. This was done for all nodes accross the interface at a particular time and the resulting gradients were then averaged to obtain an average teperature gradient at an instant of time.

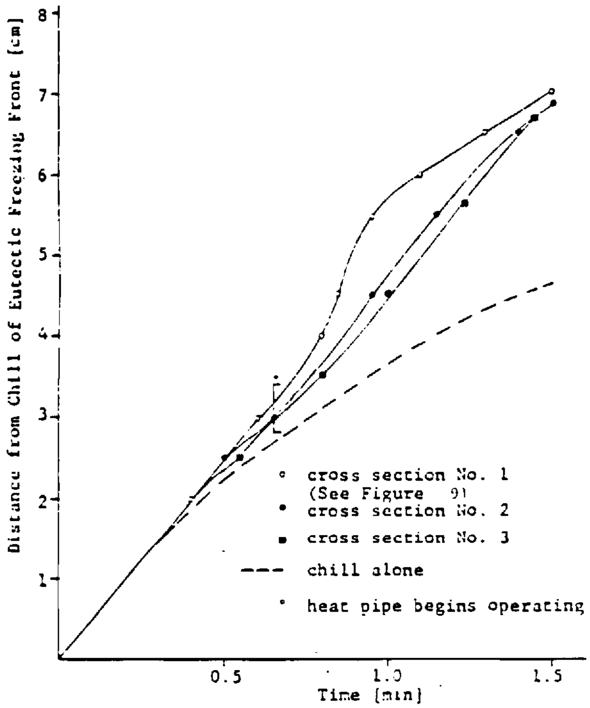

Figure 8. Distance Eutectic Freezing Front has Traveled from the Chill as a Function of Time.

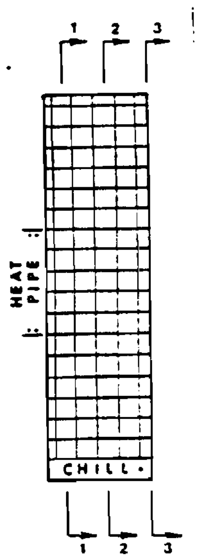

Figure 9. Vertical Cross Sections Used to Describe the Velocity of the Eutectic Freezing Front as a Function of Time.

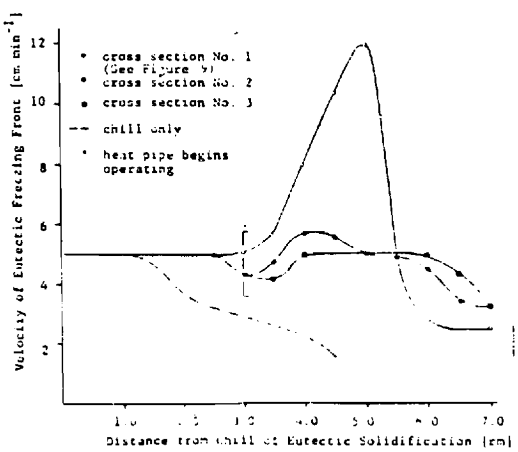

Figure 10. Velocity of the Eutectic Freezing Front as a Function of the Front's Distance from the Chill.

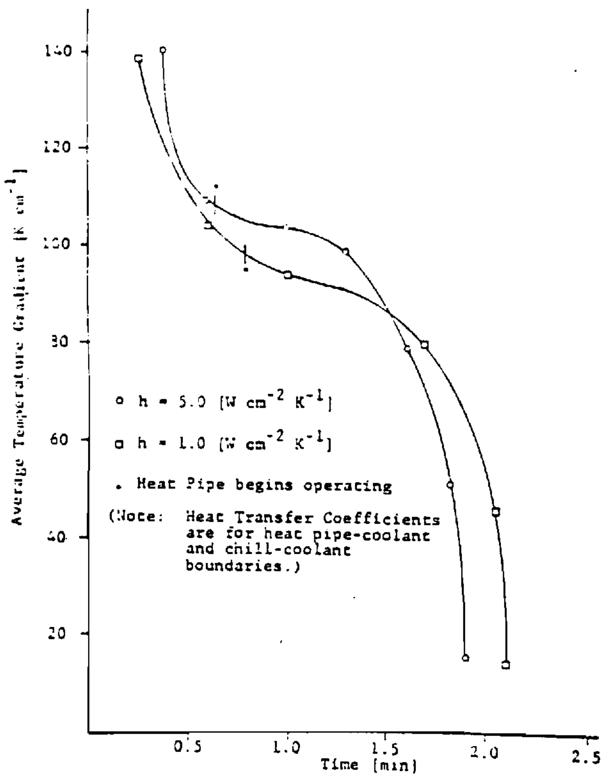

Figure 11. Average Temperature Gradient as a Function of Time for the Heat Pipe and Chill in Combination.

CONCLUSIONS

It was determined that a heat pipe may be used to materially affect the process of cooling and solidification during casting. The effects of using a heat pipe include changing the time required for casting, changing location and velocity of the freezing front, and changing temperature gradient at the front. By using a number of heat pipes of variable conductance and by activating them at various times during casting, it should be possible to greatly affect casting production and to alter mechanical properties of the parts.

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The George W. Woodruff
School of Mechanical Engineering

A COMPUTER-AIDED DESIGN SYSTEM FOR CASTINGS
PROGRESS REPORT NO. 2

PREPARED BY:
PROJECT TEAMS TASKS I, II, IV AND VII
SCHOOL OF MECHANICAL ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY

Georgia Institute of Technology
Atlanta, Georgia 30332

A COMPUTER-AIDED DESIGN SYSTEM FOR CASTINGS

Progress Report No. 2

Prepared by:

Project Teams Tasks I, II, IV and VII
School of Mechanical Engineering
Georgia Institute of Technology

Covering the Period from
April 1984 to April 1985

Prepared for:

National Science Foundation
Division of Design, Manufacturing and Computer Engineering

Attention: Dr. John Mayer, Jr.
Program Director of Manufacturing Processes
Grant No. MEA 32 11524

October 1985

CONTENTS (continued)

CONTENTS (continued)

APPENDICES

ABSTRACT

A large scale investigation of the problems associated with the computer-aided design of castings has been conducted by collaboration at two major academic institutions. This report comprehensively summarizes the work completed during the second year of a second three-year funding period at the Georgia Institute of Technology and the University of Michigan.

Seven task areas have been selected for investigation which relate to the major scientific or engineering related roadblocks in this area. These roadblocks pertain to the geometric modeling/physical simulation problems, the provision of thermal transport data, the control and prescription of interfacial heat flux during solidification, the problem of filling transients associated with pouring of castings, the modeling of interfacial phenomena, the accurate description of the interaction of the molding medium and the solidifying casting and finally the total economics of the computation system itself.

The resources of the two academic institutions have been brought to bear on these tasks over a six year funding period. The investigation has been guided by a blue-ribbon steering committee representing all aspects of the industry, its technology-related trade associations and several government agencies. The investigation should prove to have a major impact upon both the productivity and product quality of the nation's metal casting industry.

ACKNOWLEDGEMENTS

The current program is being supported by the Division of Design, Manufacturing and Computer Engineering of the National Science Foundation under Grant Number MEA 8211084. During the past year John Mayer, Jr. has served as NSF Program Manager.

The present report, which reviews progress over the second year of a second three-year program, is the result of the effort of several individuals associated with the various tasks, as well as the overall aspects of the project:

Professor J. T. Berry, Co-Principal Investigator, Tasks I & VII

Professor P. V. Desai, Co-Investigator, Task IV

Professor J. G. Hartley, Co-Investigator, Task II

Professor G. T. Colwell, Co-Investigator, Task VII

The following students participated in the experimental and computational efforts:

V. Gourisankar, Task I

P. Krishnan, Task IV

J. Moosbrugger, Task VII

S. Park, Task II

The manuscript was prepared by Tonya Moore and Melinda Wilson. Ms. Wilson also held the overall responsibility of assembling and collating the contents of the report and arranging for its distribution.

RESEARCH OBJECTIVES

A Computer Aided Design System for Castings

There are a variety of economic and technological advantages associated with the application of computer aided design to castings and casting production. The challenges involved in designing castings, including the issue of basic capability, the design of casting rigging, including provision of risers, gates, chills, and patterns, and the delivery of such designs successfully into a production system are problems which are faced in all areas of the foundry industry, including permanent mold and die casting plants.

At the outset of the current research program, a number of important scientific or engineering related "roadblocks" were recognized as being limitations to the widespread application of computer aided techniques to the solution of these problems. The principal roadblocks were identified as follows:

1. Utilizing geometrical modeling techniques and linking them successfully with simulation technologies.

2. Providing for those computations, the various thermal data required for successful simulation of both the casting and the mold behavior.

3. Evaluating associated costs of computation of the various alternative modes of simulation currently available, and providing easy access for the casting producer and designer into computer codes associated with these technologies.

4. Assessment of mold filling transients, including fluid flow and heat transfer and their interrelationships during the period of pouring and immediately thereafter.

5. Describing the various applicable boundary or interface conditions in the processes concerned and determining their order of importance.

6. Modeling those phenomena describing, for example, the interaction of thermal expansion-contraction of both casting and modeling media during freezing and feeding limitations so that distribution and magnitude of unsoundness might be predicted.

In the current program these areas have been extensively investigated by the research teams at Georgia Tech and The University of Michigan with considerable interaction between the individuals involved in each of the areas of research investigation. Considerable progress has been achieved in each of these areas and in the interfaces lying between them. These achievements are described in detail later in this report. Based on these advances, the uncovering and identification of areas where further effort can bring about a substantial forward movement in implementing computer aided design in the foundry industry has led to the continuation of research in this area. The objectives of the program involve not only an extension of several of the areas currently under investigation, but other technologies which are critical and important in the implementation and utilization of a computer based design system. Some new technologies are under investigation which involve instrumentation techniques and new solidification control technologies to further the installation of CAD in the foundry industry.

Several research objectives are being emphasized in the present study because their achievement can markedly advance CAD in this area. New research tasks to approach these challenges are incorporated in this research activity. This project is directed toward new areas and new levels of achievement necessary to accomplish the goal of implementing computer aided design of casting in the foundry industry. While some are based on current work, others are directed toward new areas to bring about the extension of computer applications in the cast metals industry.

Specifically, the objectives of the continuing research are:

1. Development of software for interfacing geometric modeling and heat transfer programs in two and three dimensions.

2. Characterizing thermal properties of current and future molding media.

3. Development of software for simulation of heat transfer in solidification of metal castings which is compatible with mini-computer systems and utilizes a simple and directly accessible input and output organization.

4. Assessing transient heat transfer in pouring, gating and mold filling. .

5. Investigating acoustic emission techniques for monitoring solidification.

6. Characterizing metal and mold wall movements and the role of thermal gradients in casting solidification.

7. Studying techniques for direct control of heat flux during casting solidification.

The program is being carried out by researchers at The Georgia Institute of Technology and The University of Michigan with a high degree of cooperation and research task interaction. The research areas 1, 2, 4 and 7 are centered at Georgia Tech, and areas 3, 5 and 6 at Michigan.

This project will continue a major program to study the scientific, engineering, and economic aspects of computer aided design in the casting industry. The implementation of CAD techniques in castings development and manufacturing production is the primary objective of this program.

Progress Report

This report summarizes progress on The Georgia Institute of Technology portion of the collaborative effort with The University of Michigan. The period covered by this report extends to April 1985. The project has been divided into seven tasks, four of which are being conducted at The Georgia Institute of Technology and are summarized in the following paragraphs (Tasks I, II, IV and VII).

The official starting date for the program was February 1, 1983.

This progress report first describes collaborative efforts between the two participating institutions. Following are reports on each of the four areas of activity at The Georgia Institute of Technology.

Collaborative Efforts

Rational for Collaboration

The program utilizes the resources of two major universities, principally the Department of Materials and Metallurgical Engineering and Chemical Engineering at The University of Michigan and the School of Mechanical Engineering at The Georgia Institute of Technology. Since the program is a multi-disciplinary one involving the areas of transport phenomena, computer aided design, computation techniques, solidification metallurgy and mold material characterization, the skills of a large number of individuals must necessarily be involved. The individuals within the two institutions concerned represent some of the most experienced researchers in the areas involved, working together with innovative younger contributors to the various disciplines represented.

The project co-directors have worked together over a number of years in the area concerned and are keenly aware of the complementary strengths at their respective institutions.

The project directors have also been able to assemble the nucleus of a steering committee from their contracts with the AFS Heat Transfer Committee. The composition of this committee has been expanded to include wider industry representation.

CADCAST

An Advisory Board has been formed and includes members of the American Foundrymen's Society Heat Transfer Committee and other representatives from various segments of the foundry industry with interests in computer aided design, as well as the U.S. Army and U.S. Air Force who have computer aided design of casting programs. A representative of the National Science Foundation also participates as a member of the Advisory Board.

A list of Advisory Board members and guests who attended the November 1-2, 1984 meeting held in Atlanta is appended, together with its agenda.

Advisory Board

Mr. Ed Borto
U.S. Army Tank-Automotive Command

Mr. A. Stubbs Davis
ESCO Corporation

Mr. Edmund M. Dunn
Arthur D. Little, Inc.

Mr. Wally Evans
Ford Motor Company

Dr. Carl H. Jacobs
Zimmer

Mr. Gerald K. Ruhlandt
General Motors Corporation

Mr. Ross Martin
FMS Corporation

Mr. F. John Mazur
Hayes-Albion Corporation

Mr. Jagan Nath
Amcast Industrial Corporation

Mr. Lee Plutshack
FOSECO

Dr. Otto K. Kiegger
Tecumseh Products Company

Mr. Michael Robinson
TRW

Mr. Ronald W. Ruddle
Consultant

Dr. S. David Sanders
Caterpillar Tractor Company

Mr. Dave Schmidt
American Foundrymen's Society

Mr. Bob Spear
Alcoa Laboratories

Mr. William Spurgeon*
National Science Foundation

Dr. Lionel J. Sully
Doehler Jarvis

Mr. Alfred E. Umble
Bethlehem Steel Corporation

Mr. Manfred Walther
Abex Corporation

Dr. Robert Warrick
The Lynchberg Foundry

Mr. Thomas Watmough
International Harvester Company

Mr. Charles A. Zanis
David W. Taylor Naval Ship R&D
Center

*As of April 1985

OTHER INVITEES - NSF ADVISORY BOARD MEETING

NOVEMBER 1-2, 1984
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA

William Andresen
Viking Diecast Corp.

Steve Antolovich
Georgia Tech

Mike Beffel
Univ. of Michigan

John T. Berry
Georgia Tech

Wayne Book
Georgia Tech

J. A. M. Boulet
Georgia Tech

Leo Buchakjian
General Electric Co.

Gene T. Colwell
Georgia Tech

A. D. Cotney
West Point Foundry

Wallace Day
Columbus Foundries

P. V. Desai
Georgia Tech

Randy Erickson
Dynarad Corp.

Peter Findlay
ADCI

Dr. Harold Gegel
Wright-Patterson AFB

V. Gourisankar
Georgia Tech

Dan Gulden
E. V. Camp

James Hartley
Georgia Tech

Jeff Heinen
General Electric Co.

Harvey Henderson
Consultant

Tim Hitchcock
Hitchcock Foundries

Thomas Hurley
Bodine Aluminum Co.

A. Jeyarajan
TRW, Inc.

Avery Kearney
Trialco

P. Krishnan
Georgia Tech

Kimio Kubo
Osaka Univ., Japan

Arno Louvo
VTT, Finland

Charles Lawery
Lockheed

William McCollum
ITT Grinnell Corp.

Carolyn W. Meyers
Georgia Tech

John Moosbrugger
Georgia Tech

Larry Nolan
McAUTO

S. Park
Georgia Tech

Robert D. Pehlke
Univ. of Michigan

Glen Reinemann
Morris Bean and Co.

Abe Scheinker
Northrop Corporation

Rod Scollard
Boeing Airplane Co.

Rudy Sillen
Novacast AB, Sweden

Roger Stafford
SDRC

Mike Stallybrass
Georgia Tech

E. E. Underwood
Georgia Tech

H. P. Wang
General Electric Co.

John White
Georgia Tech

John R. Williamson
Air Force Wright Dynamics
Lab

Ben Winter
Univ. of Michigan

CADCAST 1984

ANNUAL ADVISORY BOARD MEETING
Joint GT-UM Program in Computer Aided Design
for Casting and Solidification Technology
(Supported; by NSF)
Atlanta, Georgia

Thursday, November 1

8:30	Registration: SST Building Lobby	
9:00	Welcome	Dr. John Brighton*
	Research Progress	
9:15	Introductory Remarks	Prof. R. D. Pehlke+
9:45	Geometric Description	Dr. Toby Boulet*
10:15	Break	
10:30	Computer Code Development	Mr. Mike Beffel+
11:00	Data Base Development (i) Mold Transport Model	Dr. James Hartley*
12:00	Luncheon: Brittain T-Room	
1:30	Data Base Development (ii) Expansion-Contraction Phenomena	Mr. Ben Winter+
2:00	Forced Convection Phenomena	Dr. P. V. Desai
2:30	Experimental Techniques for Controlling and Monitoring Solidification (i) Acoustic Emission Devices (ii) Heat Pipe Technology	 Mr. Ben Winter+ Dr. G. Colwell*
3:15	Break	
3:30	Advisory Board: Communications and Discussion of Progress	

*Georgia Tech
+University of Michigan

5:00	AFS Heat Transfer Committee Meeting	
7:00	Cash Bar at Sheraton-Atlanta Hotel	
8:00	Dinner meeting at Sheraton-Atlanta Hotel 590 West Peachtree Street, N.W.	

Friday, November 2

Related Research

8:30	Cooperation with Overseas Groups	Prof. John Berry
9:00	HUBERT Project of the Nordic countries	Arno Louvo, VTT, Helsinki, Finland
9:30	Japanese Developments	Dr. Kimio Kubo Osaka University, Japan
10:00	Discussion	
10:15	Break	
10:30	NOVACAST Program - A first level software package	Rudy Sillen, Novacast AB, Ronneby, Sweden
11:15	Demonstrations/Displays TIPS/PADL - NSF Program NOVACAST - Novacast AB CATSOFT - Catronix Corporation	
12:00	Luncheon: Student Center, Room 301	
1:15	Visits A. Materials Handling Research Center B. Flexible Automation Laboratory C. Heat Pipe.Heat Transport Laboratories	
2:00	Reconvene: Summary of current industry needs	Prof. Pehlke
3:00	Closure	Prof. Berry

Publications of Georgia Tech CADCAST Group

All full listings of the publications of the CADCAST group at Georgia Tech, together with those of the University of Michigan group are listed in the Appendix A.I. which forms part of a paper to be presented to the Investment Casting Institute in Los Angeles in October 1985.

TASK I

DESIGN AND CONSTRUCTION OF A GEOMETRIC MODELER FOR CAD OF METAL CASTINGS

TASK I

DESIGN AND CONSTRUCTION OF A GEOMETRIC MODELER FOR CAD OF METAL CASTINGS

INTRODUCTION

Since the inception of the CADCAST project in 1979 there has been a great deal of progress both in the area of solid modeler development and in that of FEM codes. The early examination of a variety of solid modelers by the CADCAST team members indicated that this was an area of continuing development and that the flexibilities and general applicability of the modelers examined were continually being enlarged and updated.

Special experience was gained in handling building and weight estimation of shaped casting configurations with three systems:

TIPS-1

PADL-1

CatSoft

Although all three had some limitations in modeling complex surfaces, several simple cast shapes with rigging attached were modeled without difficulty. One of the major limitations concerning most modelers at the beginning of the last grant period (in 1983) was the lack of connectivity with currently available FEM or FDM codes.

The TIPS-1 package did include an enmeshment routine for 2-D (FEMNET) which was of a semi-automatic nature, but some difficulties were envisaged in its use. In the interim, a study was commenced (Dalton, ref. 1) which was aimed at demonstrating how such an enmeshment routine might be constructed.

The starting point for the study was a csg based solid modeler of T. Kohler (CatSoft) and the FEM code chosen was co-investigator J. G. Hartley's FE-2D transient heat transfer program.

As will be seen from the following summary, the routine constructed - AIDE - successfully demonstrated how a solid model could be built, sectioned, enmeshed, numbered and then passed to the FEM program where several simple 2-D heat conduction problems were solved.

Since the publication of the thesis, several commercially developed two - and three - dimensional enmeshment routines have been made available, which also permit the interactive semi-automatic enmeshment and node numbering within boundaries provided ;by solid modelers. Among these combinations are:

GEOMOD-SUPERTAB (SDRC)

CAE*PAC (CAE*TEC)

GMSOLID-SMUG (GM CORPN)

The first combination is capable of running on typical mini-computer-based systems (VAX11/780 etc) as is the third. The second package is of particular interest since it will run on Apollo and IBM PC/XT type machines. It should be noted in connection with the second package that at the present time the execution of the FEM code would be undertaken on a minicomputer after having completed the solid model building and enmeshment etc. on a smaller machine. The interface currently provided enables the user to run computations using NASTRAN or MARC codes for example.

Summary of Contents of AIDE Interfacial Package

The AIDE package was constructed to link together the coarse 2-D mesh, which can be generated automatically by the CatSoft modeler [2], and the transient heat conduction program FE-2D [3].

With the CatSoft modeler the enmeshment process superimposes a fixed grid of squares over any selected cross section.

Squares falling wholly inside the section are interior elements, while squares falling partially inside are boundary elements. The shapes of these boundary elements are determined by the polygons that result when surface points, those at which grid lines intersect the object boundary, are connected by straight lines. Undesirable polygons result in some cases, either because of grid line coincides with the object boundary or because large corners of the section are lost.

The case of grid line coinciding with the object boundary is unlikely, but can occur. It is unacceptable because CatSoft requires the mesh to have a "skin" of boundary elements. If a grid line coincides with an object boundary, an ill-defined boundary element will result. These elements can be detected because the proper color boundary lines will not appear when the mesh is displayed on the screen.

The second case, that of corners being lost, is a more frequent problem. Sharp corners will always be chopped off because of the way, discussed above, that boundary elements are created. This problem can be minimized by making the amount of corner loss as small as possible.

Since undesirable polygons result in some cases, it is advantageous for the user to verify the accuracy of the mesh by viewing it on the screen. If any problems are detected, they can be eliminated by changing the size or position of the section, then generating the mesh again. Although some user input may be required to obtain an accurate mesh, this process is considerably faster and much less prone to error than generating a mesh by hand. Mesh data are stored by CatSoft in the direct-access file MESH.TDB.

Processing the Mesh

The mesh data file generated by CatSoft requires processing before it can be used by the finite element thermal analysis program. The interfacial program called AIDE had been written to perform this processing. AIDE is composed of three subroutines: FETCH, BAND and DRAW.

Subroutine FETCH retrieves mesh information from file MESH.TDB for each element in the object. It numbers the nodes consecutively and keeps a record of which colors are present. The "colors" are utilized to denote boundaries. Elements generated by the modeler are polygons with three to six sides, but the finite element program FE-2D requires triangles exclusively. FETCH divides each polygon into suitable triangles and numbers each triangular element consecutively. Results from FETCH are written to second direct-access file called MESH.DAT. Also, a list of the node numbers that belong to each element is stored in an array to be used in subroutine BAND. Details of the algorithm used in FETCH may be found in the Dalton thesis.

The method of finite element analysis involves solving large numbers of linear simultaneous equations. There are a variety of numerical techniques available for solving such equations, but finite element programs normally employ banded-matrix solutions. These solutions are preferred because they greatly reduce space requirements in memory and speed the execution of the analysis. Banded solutions take advantage of matrix sparsity by clustering entries along the diagonal, thus forming a non-zero "band" of entries. Only entries within the band need to be stored in memory and used during execution. Hence, the width of the band should be made as small as possible. Since the position of a given nodal entry in the coefficient matrix depends on the node number, the bandwidth can be optimized by renumbering the nodes. Subroutine BAND renumbers the nodes efficiently so that the bandwidth of the coefficient matrix is reduced. The algorithm in BAND is taken primarily from a published program written by R. J. Collins [4]. The new node numbers generated by BAND replace the old node numbers in file MESH.DAT.

Subroutine DRAW simply reads the mesh information from file MESH.DAT and displays a picture of the new mesh on the graphics screen.

Performing the Thermal Analysis

As was mentioned, an existing finite element program, FE2D, was chosen to perform the thermal analysis. The original version of FE2D was written by Dr. James Hartley of the Woodruff School of Mechanical Engineering at the Georgia Institute of Technology. In order to improve compatibility with the CatSoft database and the Terak computer utilized, a modified version of FE2D was written.

An additional routine named Quiz was also added to accept and process user input, and certain changes were also made to FE2D itself. A main program named THERM was written to call the two routines QUIZ and FE2D.

Subroutine QUIZ interactively accepts input from the user. All data generally applicable to the mesh in the current analysis are already stored in file MESH.DAT, so QUIZ does not require any additional input concerning the geometry of the object. It does require all the information needed to simulate particular thermal conditions. QUIZ asks for descriptive title of the current problem and whether the problem is steady-state or transient. If transient, QUIZ asks for the maximum time of interest, how often results should be printed, and the initial temperature of the object. It inquires about the thermal conductivity and specific heat of the material. Finally, it asks for the boundary conditions represented by each color present on the boundary of the section.

In order to guard against input errors by the user, QUIZ repeats a question if an invalid response is given. Also, QUIZ echos the response back to the screen for the user to validate. If the user detects an incorrect response to a previous question, he may back up as many questions as necessary by entering zeros for responses to the current question. After all the required input is obtained, QUIZ calls a routine named STORE.

STORE writes the problem title, number of nodes and elements, material properties, and node coordinates to the output file names ANS.,DAT. It also stores boundary condition information in file MESH.DAT, but does not disturb the mesh data already located there. This storage is significant because additional runs of THERM may be made without executing AIDE again.

Thus the user may experiment with different thermal conditions without reprocessing the mesh each time. After all necessary information is stored, control is passed to the FE2D routine.

Finally it should be noted that the AIDE package was constructed to examine the problems of 2-D enmeshment and to determine the feasibility of linking with a finite of element code. The suite of packages concerned in its present form has some distinct limitations; therefore, it cannot for example handle temperature dependent material properties and in addition modifications would also have to be made for latent heat liberation.* It could however be utilized for cooling problems in the solid state and other heat conduction problems in the foundry. The modifications necessary to be able to handle solidification would also necessitate running the total package on a computer larger than the TERAK (128 kilobytes of memory) upon which the original package of software was run.

1. B. B. Dalton, "Integrated Computer System for Thermal Analysis of Castings," Master's Thesis presented June 1983, Woodruff School of Mechanical Engineering, Georgia Institute of Technology.

2. T. Koehler, CatSoft program produced for Catronix Corporation, Atlanta, Georgia.

3. J. G. Hartley, FE-2D program for transient heat conduction study.

4. R. J. Collins, "Bandwidth Reduction by Automatic Numbering," Intl. Jnl. for Numerical Methods in Engineering (12) 1973, pp. 345-356.

*NOTE:

The FE-2D program referred to in this section of the report has since been modified to accommodate the liberation of latent heat and has been applied to a problem of casting feeding, see Appendix A.II.

TASK II

CHARACTERIZATION OF TRANSIENT EFFECT IN CURRENT AND FUTURE MOLD MEDIA

SUMMARY OF CURRENT STATUS

TASK II

CHARACTERIZATION OF TRANSIENT EFFECT IN CURRENT AND FUTURE MOLD MEDIA

SUMMARY OF CURRENT STATUS

The goal of Task II is to provide information regarding the thermal behavior of mold materials for use with the simulation of casting solidification. Current efforts are concentrated on the development of a predictive model for the thermal conductivity of bonded molding sands and the analysis of the heat and moisture transport in green sand molds.

Previous results were summarized in Progress Report No. 1 (November 1984) as follows:

1. A new model for the thermal conductivity of two-component systems has been developed and verified experimentally. This model is semi-empirical in that three experimental measurements are required to determine model parameters which account for sand particle size, shape, and size distribution. Predicted thermal conductivity values for two-component systems obtained with this model are within about two percent of experimental values.
2. The two-component, thermal conductivity model can be extended for use with three-component systems (e.g. bonded sands), but the density and effective thermal conductivity of the bonding medium must be determined.

3. Improvements in the design of the high-temperature thermal conductivity probe and in sample preparation have been made.
4. Experimental measurements of the density of western and southern bentonite particles have been completed.
5. An experimental procedure for determining the density of the porous structure formed by bentonite bonds in bonded sands has been developed. With this method, the relationship between water-clay ratio and the density of the porous structure has been studied.
6. The thermal conductivity of dense, dry bentonite samples has been measured using the hot-wire method, and work has begun on developing the procedures necessary to determine the effective thermal conductivity of the porous structure formed by the bentonite bonds.
7. An analytical model for the heat and moisture transport in green-sand molds has been developed.

During much of the period covered by this report, the doctoral student who has been involved with analytical and experimental work related to the effective thermal conductivity of bonded sand was on a leave of absence. He returned to resume and complete his research in June 1986. Therefore, detailed descriptions of the experimental program and recent developments will be provided in the forthcoming final report, and only brief statements of progress since the last report follow:

1. A three-component system model has been completed and can be used to predict the effective thermal conductivity of bonded molding sands at room temperature with very good accuracy. Further validation of the model through comparison with experimental measurements is currently in progress.
2. An empirical method has been developed to predict the thermal conductivity of the bentonite bond in bentonite-bonded sands.
3. High-temperature, effective thermal conductivity measurements on bonded sands are also in progress. This work is being done to further perfect the measurement techniques, to provide additional data for numerical modeling, and to expand the predictive capability of the three-component model to include the effects of temperature.
4. An experimental apparatus for green sand studies has been designed and constructed. Preliminary measurements of the thermal response of a green sand mold (after pouring a liquid metal into the mold) have been obtained.

TASK IV
THERMAL CONVECTION DURING FILLING

TASK IV

THERMAL CONVECTION DURING FILLING

The progress on the research on this task is summarized in the form of a paper which was presented at the 1985 Annual Meeting of the American Foundryman's Society. The paper entitled "The Thermal Performance of Gating Sprues in Sand Casting Systems" was written by P. V. Desai, Associate Professor, K. V. Pagalthivarthi, doctoral candidate, both in the Woodruff School of Mechanical Engineering at Georgia Tech and J. T. Berry, Professor and Chairman of the Department of Metallurgical Engineering at the Univesity of Alabama, Tuscaloosa, Alabama. The entire content of the paper is presented here.

ABSTRACT

A two-dimensional analysis of the heat transfer problem in gating sprues in sand casting systems is studied. The cases of constant cross-section and linearly varying cross-section of the sprue are examined. The solution for the "conjugate heat transfer problem" with varying flow velocity is determined by using a fixed finite element mesh to represent the domain. A fictitious layer of elements is used at a mold-metal interface. It is shown that this enables the exact description of the initial temperature fields in the metal and the sprue wall and obviates any assumptions of the heat transfer coefficient at the interface.

The effects of gravity, the relative thermal properties of the mold and the metal, and the varying velocity and sprue geometry are discussed. The role of convective heat transfer in determining the interface temperature for different sprue geometries is explained through numerical examples. It

is shown that steady state is reached rapidly near the exit section of the sprue. Numerical examples are presented for pure aluminum and zinc and a ductile iron flowing in silica-sand sprues.

INTRODUCTION

During the past three decades significant advances in the thermal design of gating systems have been brought about by a concerted effort to apply the fundamental principles of heat transfer and fluid mechanics to foundry problems. Advances in computational fluid mechanics and heat transfer have allowed one to examine complex mathematical models of the solidification processes without making over-simplifying assumptions. Some of the major problems associated with improper design of gating system runner, down-sprue, and so forth, are 1) aspiration of gases leading to gaseous unsoundness; 2) excessive heat loss leading to a loss of fluidity of the metal and hence its non-uniform distribution within the casting cavity; 3) premature solidification in a non-sequential fashion; 4) slag and dross collection in the molten metal; and 5) excessive turbulence in the gating passages.

Considerable attention continues to be focused on these problems. Extensive work has not yet been done in the area of heat loss from the molten metal to the gating system and attendant problems. In other words, a comprehensive examination of the heat transfer process in the gating system is necessary to obtain better estimates of the temperature loss in the gating system. A more rigorous mathematical and computational formulation than that available in the literature should improve the prediction of heat losses within the gating system. With such knowledge, it should be feasible to accurately assess the degree of superheat of the poured melt required to maintain an adequate fluidity of the metal before it enters the casting cavity.

The main aim of the present work is to establish suitable design criteria for the thermal performance of two-dimensional or axisymmetric gating sprues in sand casting systems. A complete solution of the heat transfer problem requires the determination of the temperature distribution in the mold wall and both the flow field and temperature distributions in the flowing metal. A prior calculation of the constant (with respect to time) non-uniform velocity field enables one to focus on the solution of the thermal energy equation. Thus the problem reduces to a transient conjugate heat transfer problem requiring a simultaneous calculation of the temperature fields in the mold and in the metal.

An analytical solution is very complex due to the conjugate nature of the problem with unknown time-dependent boundary conditions at the mold/metal interface. A numerical solution via the finite element method needs to be used to solve the problem.

Due to the low Prandtl number of liquid metals, a slug flow assumption may be made. The sprue may be assumed to be instantaneously filled since the main emphasis is on studying the heat transfer characteristics of gating sprues. In a fixed finite element mesh which describes the domains of interest, very high initial thermal gradients at the interface may be smoothed out by using a fictitious layer of elements having a low thermal capacity and a high thermal conductivity (in the direction of maximum heat flux).

This paper presents a solution of the problem for several mold-metal combinations, horizontal runners and vertical down sprues, several taper angles of the sprue, and for several inlet Peclet numbers. The thermal diffusivity ratio of the mold and the metal and the extent of taper and time (in the form of a Fourier number) are the important input parameters that govern the solutions. The effects of these on the interface temperature have been studied and presented in the form of graphs usable by the practicing foundry engineer.

RESEARCH ON HEAT LOSS TO SPRUE WALL GATING SYSTEM

In a major study to identify some of the factors that determine temperature loss in a runner [1], it was assumed that the 1) flow of metal was turbulent in the passage; 2) differences in thermal conductivity of different metals did not significantly affect temperature loss; 3) flow rate remained constant; 4) mold temperature remained uniform; 5) time scale should be measured from the time the first metal left the runner. Metal temperature may be assumed not to change until that time; 6) shape of the cross-section of runner may be disregarded, only the product of perimeter and length being pertinent; and 7) the latent heat effects due to freezing and remelting were negligible.

In that work, the one-dimensional heat conduction equation and a simple energy balance across the mold-metal interface were considered for an infinite quarter plane. Contact resistance was not included. Plots of temperature loss vs. time were obtained with the critical ratio (defined as surface area/flow rate) as a parameter. The method could, to a first approximation, be used to predict temperature loss for sand molds. In the case of metallic molds this would be less valid since the method assumed instantaneous filling of the runner and no heat loss during this period. For short times after filling of the runner even this method predicted freezing. It was suggested that subsequent hot metal remelted the first frozen layers.

In a modified method used to account for the initial filling transient, a friction factor was introduced at the mold-metal interface to compute the velocity of the moving free surface [2]. It was shown that the initial transient velocity was much greater than the steady flow velocity when the mold was filled. This was considered to alleviate the temperature loss from the metal at initial contact. In the analysis, a sand-metal interface resistance was considered; the domain was considered as a semi-infinite region, thus ignoring the curvature of the runner. This assumption was supported by the argument that for sand molds (with a low thermal conductivity) the temperature changed only over a small distance. The effect of metal velocity was included as a convective term by writing an energy balance for the mold/metal. In order to obtain an analytical solution a small temperature loss assumption was made. The problem was solved for the cases of constant flow velocity and linearly decreasing flow velocity.The temperature loss was found to decrease with time but was proportional to the residence time of a fluid element in the runner. Additionally, an electrical analog system was also built to verify the computational model. The heat loss equation developed by these methods was applied to study such system characteristics as temperature loss vs. runner length, flow rate, runner diameter, pouring temperature, and mold thermal properties [3].

From a similar treatment of the problem of heat loss from liquid metal to walls of the runner, it was additionally observed how the fluidity of aluminum was altered by surface tension effects present in small diameter tubes or channels [4]. Vibration in a favorable direction was shown to increase the ability of metals to flow into small diameter channels.

In a study on the effect of alloy composition and mold material in determining flow-ability of metals, it was found, by plotting graphs of flowing capacity of aluminum alloys vs. length of channel, that sand molded channels exhibited a critical channel length which caused an abrupt increase of the flowing capacity [5]. This critical length was composition sensitive and was not found in case of steel molded channels. The studies showed that direct correlations between flowing capacity values and fluidity values may not exist in general. Thus it was suggested to use "graphs of flow" rather than standard fluidity tests to predict flow of metals in gating systems.

In a subsequent experimental work, it was reported that the cooling experienced by a molten metal along the sprue of a permanent mold may be considerable [6]. The effects of superheat on the flowing capacity of a metal in permanent molds were found to be more significant in case of long freezing range alloys, e.g. Al-4.5% Cu, and much less in case of 99.99% pure Al. It was suggested that flow possibilities may be affected at low pouring temperatures due to possible formation of fine crystals at the entrance to the channel. However, it should be noted that these results are relatively invalid for insulating mold materials such as sand.

In a later study the flow possibility of aluminum in sand molded channels was examined [7]. The main concern in this work was the relative significance of ratio of cross-sectional area to perimeter of channel (S/P) on fluidity and also the critical channel length below which obstruction due to solidification did not occur. As in the previous cases, the work was largely based on graphs derived from extensive experimentation with different alloys in sand gates. In addition, however, a theoretical basis for fluidity of metal was provided. An analytical expression for the critical channel length, L_c, including metal/mold properties and the S/P ratio, was given to show that L_c varied as the square root of the time for completion of obstruction process near the open end of the channel. It should be noted that in spite of these suggestions, fluidity and metal flowing capacity are considered interchangeable variables.

About a decade ago a simplified analysis of a metal flow down a long metal cylinder was presented [8]. For this case, heat flow is interface controlled. Therefore, the radial temperature gradients in the mold and the metal were ignored. By neglecting axial heat conduction along the interface, a simple heat balance was written for the interface. The temperature loss was assumed small to solve the resulting first order ordinary differential equation. However, the method assumed a prior knowledge of interface resistance from experimental or other sources, limiting its applicability.

In yet another study the transient mold filling process for bottom-gated vertical thin-walled castings with linear laminar and turbulent flow patterns was examined [9]. The heat transfer calculations were performed by assuming the presence of a thin mold wash. It is easy to simulate the case without a mold wash since there is always some contact resistance. Even the presence of an air gap could be readily considered with this method. IN the analysis, heat flux from metal to mold was determined via an effective thermal resistance for the mold/mold-wash/metal combination. A simple heat balance for an elemental volume and an integration over the entire time of interest were used to calculate the temperature loss during filling. The unknown stream temperature at the moving free surface was assumed to vary quadratically across the section. Here again, axial conduction was neglected and the interface resistance was assumed known from prior experiments. The analysis was essentially one-dimensional, with the mold temperature assumed as constant. Except for the mathematical complexity, therefore, the method was very similar in assumptions to the earlier works.

Transient cooling of liquid in tubes under laminar flow conditions was studied recently by using an integral transform technique [10]. The outside of the tube was considered to be at a fixed temperature, maintained by convective cooling. The distance the liquid could travel before beginning to freeze was calculated. Although attractive, the method is not strictly applicable to gating systems in sand molds since it assumes negligible resistances for the tube wall and for the interface.

On the basis of the derived equation, it is possible to calculate the actual fluidity of the metal for a given amount of superheat, and conversely, if the temperature loss in the gating system is known, it is possible to calculate the fluidity of the metal that enters the casting cavity [8]. It is simple to determine if, for a given temperature loss, the metal has adequate fluidity or not. This is because the minimum required fluidity of a metal to completely fill the mold depends only on the mold characteristics and is independent of casting conditions such as temperature, pouring condition, etc.

The methods of predicting temperature loss described in the preceding have used many simplifying assumptions. To validate these assumptions it is necessary to examine the fundamental nature of the forced convection heat transfer process. Many physical situations do not satisfy these simplifying assumptions and, therefore, have to be examined by a more rigorous thermal analysis.

The cooling of flowing molten metal by surrounding mold walls is an internal forced convection transient heat transfer problem that requires a simultaneous determination of both the temperature fields in the mold and the metal as well as the flow field in the metal. Momentum diffusion in liquid metals is insignificant relative to the corresponding heat diffusion. Hence, good practical results may be obtained by examining the energy equation separately. In the present work the velocity field in a tapered rectangular sprue is determined by using the continuity equation alone. The assumption of instantaneous filling ensures the applicability of the continuity equation.

The temperature fields may be calculated by solving the total energy equation for the two media simultaneously. In a comprehensive survey of earlier work done in solving conjugate heat transfer problems, it was pointed out that most of the methods in the past assumed a known temperature or heat flux at the mold metal interface [11]. For example, in some analyses the transient conduction equation was solved for the sand with an interface condition derived from the heat balance of an element, and the total temperature loss in the metal was considered to be a small fraction of the excess temperature, $T_{pouring} - T_\infty)$[2,3]. These assumptions enabled the determination of the temperature change of the bulk fluid along the flow direction. This renders the heat transfer in the flow process to be one-dimensional in nature. Velocity and temperature was assumed constant across a section and heat conduction transverse to flow direction was alone considered significant. The difficulty in this type of analysis is the determination of the actual heat transfer coefficient at the interface since the latter depends on changes (with respect to the time) of flow rate, inlet peclet number, relative thermal diffusivities across the interface and so forth.]

The conjugate heat transfer problem of a liquid metal in an initially empty channel at a cooler temperature, recently solved by using a slug-flow assumption and a deforming finite element method, used a fictitious layer of elements at the interface between the mold and the metal to smooth out the large temperature gradients at initial contact [11]. The flow velocity was assumed constant.

In the thermal design of sprues in sand molds, the problem is similar to the conjugate heat transfer problem. Since the sand mold has poor thermal conductivity, the sprue may be assumed to be instantaneously filled, in predicting the overall temperature loss. The absence of an advancing free surface makes it unnecessary to use a deforming mesh. In other words, an Eulerian description of the problem is adequate. A fixed finite element method can be used to compute the temperature fields in the mold and the metal. The finite element equations are derived in this work on the basis of the total energy equation for the metal and the mold. The temperature distribution along the wall is a solution of the problem and not an assumption. In other words, no assumption is made regarding the heat transfer coefficient at the mold-metal interface. It is interesting to note that the method is quite general and can be applied to a variety of sprue shapes or to other parts of the gating system. Although the actual problems solved is for a rectangular cross-section, it can readily be extended to axisymmetric domains. The solution and the method are independent of the actual magnitude of the temperature loss.

MATHEMATICAL ANALYSIS AND NUMERICAL SIMULATION

The geometry for the proposed problem is illustrated in Figure 1. The solid sprue wall, region R2, is assumed to be of considerable thickness. Liquid metal, region R1 is assumed to fill the sprue instantaneously at a temperature equal to the pouring temperature, T_p. Initially the sprue wall is taken to be at the ambient temperature, T_∞. The governing equations for the system may be summarized as [12]

Continuity $$A_a v_a = A_y v_y \,, \qquad (1)$$

Energy equation for mold, region R2

$$\rho_2 C_{p_2} \frac{\partial T_2}{\partial t} = \frac{\partial}{\partial x} \left(k_2 \frac{\partial T_2}{\partial x}\right) + \frac{\partial}{\partial y} \left(k_2 \frac{\partial T_2}{\partial y}\right), \qquad (2)$$

and Energy equation for liquid metal, region R1

$$\rho_1 C_{p_1}\left(\frac{\partial T_1}{\partial t} + v \frac{\partial T_1}{\partial y} \right) + \rho_1 v\left(v \frac{\partial v}{\partial y} - g\right) = \frac{\partial}{\partial x}\left(k_1 \frac{\partial T_1}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_1 \frac{\partial T_1}{\partial y}\right). \quad (3)$$

The relevant boundary conditions are, for $t > 0$

$T_1 = T_p$ at $y = 0$ and $T_1 = T_2$ along the mold/metal interface

$(k_1 \nabla T_1) \bullet \underline{n}_1 = (k_2 \nabla T_2) \bullet \underline{n}_2$ along the mold/metal interface

$\frac{\partial T_1}{\partial x} = 0$ at $x = 0$

$$\int_0^{d_e} [k_1 \partial T_1 . \partial y] dx = \int_0^{d_e} h(x)(T_1 - T_\infty) dx \text{ at } y = L .$$

$\frac{\partial T_2}{\partial y} = 0$ at $y = 0$ and $y = L$ $\quad$ $\left. \begin{array}{l} T_2 \to T_\infty \text{ for } R_2 \\ \text{as } x \to \infty \end{array} \right\}$

The initial conditions are

at $t = 0$ $\quad T_1 = T_p$ for R_1 and $T_2 = T_\infty$ for R_2 . (5)

An analytical solution of the mathematical model is difficult due to the changing interface condition. At time, $t = 0^+$, when bulk liquid contacts the solid wall, a sudden high thermal gradient is developed at the wall. Any suggested solution method must account for the thermal contact during initial time without giving spurious oscillations. A finite element numerical solution scheme is utilized to solve the problem. The fact that the discrete finite elements can be of arbitrary shape makes it easier to adopt the finite element technique to model the interface in thermal contact problems. It has been shown that the use of a fictitious layer of thin elements of very small thermal capacity and very large thermal conductivity obviates the difficulties met with during initial thermal contact [13]. Further, the energy conservation principle is obeyed with an acceptable accuracy. In addition, the use of different size elements in different regions of the domain poses no extra difficulty in the finite element method.

In view of these reasons, the present problem is examined by the finite element technique using a thin layer of fictitious elements separating the mold and the metal. The velocity field is first calculated to satisfy continuity of flow. This is used in the computational scheme for solving the energy equation. Figure 2 shows a schematic of the finite element mesh used in the numerical simulation. A total of 8 bilinear elements are used in the direction of flow and 42 layers (24 in sprue wall, one fictitious layer and 17 layers in the liquid metal) are used perpendicular to the flow. The element size (Δx) is increased progressively as the distance from the interface increases. In the discussions that follow the taper refers to that in one half of the symmetric sprue.

INFLUENCE OF MAJOR VARIABLES ON TEMPERATURE FIELD

For practical flow rates in modern foundry practice, the contribution of the gravity and acceleration terms in the energy is four to five orders of magnitude smaller than the thermal terms. For practical sprue geometries the metal velocity seldom exceeds 25 cm/s at the inlet. For small tapers, the potential energy term is larger. Therefore, the two terms together (or separately) make no significant contribution. However, the changing velocity also affects the convective heat transfer in the sprue. This suggests separate examinations of the influence of gravity and the velocity-variation. In Figure 3, the interface temperature for aluminum in a constant cross-section, silica-sand sprue is shown. Despite a high flow rate, there is practically no influence of gravity (the $\rho v g$ term) on the interface temperature. In fact, less than a tenth of a percent error is incurred by excluding the effect of gravity in vertical sprues. Figure 3 also shows a much smaller flow rate of 8 cm^3/s (per unit thickness) but a tapered sprue (3:20). Once again, there is practically no effect of gravity on the interface temperature. One major difference between the two cases in Figure 3 is that in the latter, the temperature of the metal progressively increases downstream and is a maximum (excluding the inlet) at the exit. This may be attributed to the increasing velocity of the metal along the tapered sprue. This higher velocity of the metal near the exit section carries the metal faster than at upstream locations. This allows less time for heat exchange for the metal near the exit section. An alternative argument may be made on the basis of the energy balance equation. The heat conduction at any two sections down the sprue length may, as a first approximation, be assumed to be invariant. The gravity term $\rho v g$ has already been shown to be insignificant in determining the temperature field, at least for practical lengths of sprues. Therefore, only the convective heat transfer term and the kinetic energy term are relevant to this discussion. At the exit section the velocity is larger than at the upstream locations. This implies a larger normal temperature gradient at upstream locations. In other words, T_1 increases in the downstream direction, except for very small or zero tapers.

Although Figure 3 is shown for the initial period of time, the trend of results and discussions hold in general. As time increases, axial heat conduction tends to stabilize the transfer processes and transfer, slowly, the minimum temperature to the mid-section for slow flow rates (cf. later discussion of steady state condition and Figure 5).

The fact that the temperature gradient decreases in the flow direction initially time may be effectively used in designing gating sprues. Provision of suitable tapers may ensure acceptable levels of heat loss and hence metal fluidity until steady state is reached. Of course, there is a practical limit to which the taper could be increased. Beyond this limit, factors such as sand erosion, excessive turbulence, and dross collection may determine the performance criterion of the sprues.

Relative Thermal Properties of Sprue Wall and Metal

The interface temperature depends on the thermal diffusivity ratio, α_1/α_2, for a specified metal flow rate and sprue geometry. In this work the flow of zinc, aluminum and ductile cast iron in silica sand sprues are studied. In the present computations a superheat of 50C has been used. For this superheat and an ambient temperature of 20C, the dimensionless melting temperatures may be directly ascertained. It is evident from Figures 4a and 4d that a larger superheat than 50C would be required to avoid solidification in the case of ductile iron. Indeed, in all but the heaviest sectioned ductile iron castings superheats well in excess of this are used in practice.

Figures 4a, 4b and 4c show the dependence of interface temperature on the relative thermal diffusivity of the sprue wall. The graphs represent the interface condition after the initial material surface at the sprue-entrance has traversed 25% of the channel length. Beyond this time, the thermal gradient decreases rapidly, especially in the lower half to the sprue. All three figures are valid for a metal flowrate, Q, of 80 cm^3/sec-cm; however, the sprue-taper is progressively increased. In the case of untapered sprues (Figure 4a) it is found that a higher interface temperature results when using a metal such as aluminum with a high thermal diffusivity. This is explained on the basis that the relatively lower thermal diffusivity of the sprue wall resists propagation of heat in it, maintain a higher interface temperature. This result is found to be true for other practical flow rates as well. The introduction of a taper increases the metal velocity in the downstream direction. For a taper of 1:10 (Figure 4b), the effect of thermal diffusivity is reversed and for a 3:20 taper (Figure 4c) this reversal is even more pronounced. In other words, for these two cases the interface temperature is a maximum for ductile cast iron and minimum for aluminum, especially near the exit of the sprue. This surprising result needs closer examination. Although aluminum has the highest relative thermal diffusivity, it also has the lowest thermal capacity (ρC_p) among the three metals, viz, aluminum, zinc, and ductile iron. In an untapered sprue the velocity remains constant implying that convective heat transfer (the term, $\rho_1 C_{p1}\ v\ \partial T_1/\partial y$ in the energy equation) is determined by the y-derivative of the temperature field which is approximately constant away from the sprue-entrance. Hence heat conduction across the interface, determined by the relative thermal conductivity, plays the dominant role. However, in a tapered sprue the additional factor of convective heat transfer in the direction of flow has to be considered. The higher the thermal capacity (ρC_p), the greater will be the heat flux via convection along the flow. This introduces a competition between heat diffusion by conduction into the sprue wall and heat convection along the flow. IN the case of ductile iron, the convective term is the maximum of the three. Hence, there is less thermal energy available in the metal at any cross section to transfer into the walls. This results in a higher interface temperature for metals with a higher thermal capacity than those with a lower thermal-capacity, such as aluminum, provided that the taper is significant. In fact, for taper angles smaller than a crucial value, the conduction phenomenon should govern the interface temperatures and results similar to Figure 4a should be obtained. Figure 4d shows the results for a small taper of 1:50.

It is apparent that a faster metal flow rate tends to give higher temperatures along the interface after the same duration of time. This result has been also shown for constant cross-section sprues [13]. Figure 5 establishes this point for a 1:10 tapered sprue using zinc and silica-sand.

From the preceding discussion, it may be concluded that the effect of the thermal properties of the sprue wall material and the metal cannot be independently discussed. The simultaneous effect of sprue geometry and flow rate must also be considered.

As mentioned earlier, after a certain time (depending on the flow rate and sprue geometry) the temperature gradients decrease sharply and a steady state condition is approached. This effect is brought out in Figure 6, where the interface temperature is plotted as a function of time for three different normalized distances (Y/L) for the flow of aluminum in silica-sand. Steady conditions are approached nearer the exit section earlier than at up-stream locations. In general, with increasing time, the amount of temperature loss experienced by the metal decreases. This result was indeed observed earlier [1].

In addition to the factors discussed in the preceding, the total surface area of the sprue plays an important role in the heat transfer process. The presentation of results in the form of Figure 4, 5 and 6 would require several graphs of this type in order to include the effect of all major parameters. Even so, interpolation between graphs would be required for intermediate values of the parameters. It is more meaningful to obtain the results if some or all of the parameters are suitably combined. The combined parameter should, of course, have physical significance. Work is currently in progress in formulating such a parameter.

CLOSING REMARKS

Three categories of conclusions may be derived from this work. These pertain to 1) the solution methodology; 2) the physical nature of the problem; and 3) the application of the technique developed to actual gating systems.

The finite element method, using a fixed mesh, is very suitable to determine the two dimensional temperature field in the metal and the sprue wall made of sand. It is noted that a) the use of a fictitious layer is necessary to prescribe the highly different initial temperature distributions in the contacting media, viz. the sprue wall and the metal; b) in calculating the temperature loss, the fictitious layer obviates any assumptions about the heat transfer resistance at the interface; and c) the thermal penetration depth is important in determining the mesh size.

The numerical results using the finite element formulation provide valuable insight to the heat transfer processes in gating sprues. In particular, it has been found that a) the convective heat transfer in gating systems may not be insignificant; b) there are competing effects of heat transfer by conduction normal to the direction of flow and by convection along the direction of flow. Thus, both the thermal conductivity and the thermal capacity of the contacting media play a role in determining the temperature fields in the two media; c) it is inappropriate to extrapolate results of untapered passages to tapered passages, particularly if the taper angle is large; d) for large time-durations (of the order of the filling time of the sprue or large) the temperature gradients within the flowing metal are very small and the temperature fields tend to a steady state; and e) axial heat conduction may alter the temperature distribution along the interface, particularly if the liquid flow rate is small.

The quantitatively ascertained effects of such major variables as the liquid metal flow rate, the sprue geometry and the thermal properties may be readily applicable to industrial problems. In particular a) it is found that a higher flow rate of the liquid results in a smaller temperature loss; b) the magnitude of the thermal energy of the system is much larger than that of the mechanical energy. The effect of gravity is found negligible; c) for small tapers the thermal diffusivity ratio, α_1/α_2, is more significant than the thermal capacity of the liquid metal. For large tapers these effects are reversed; d) temperature loss decreases with time until steady state conditions are reached. On initial contact, the first liquid may even freeze, to be remelted by subsequent flow of the hot metal; and e) the major variables controlling the heat transfer process may possibly be combined together in the form of a single parameter. The numerical value of such a parameter will determine the interface temperature at any specified location. Future work will be directed toward formulating such a parameter.

ACKNOWLEDGEMENTS

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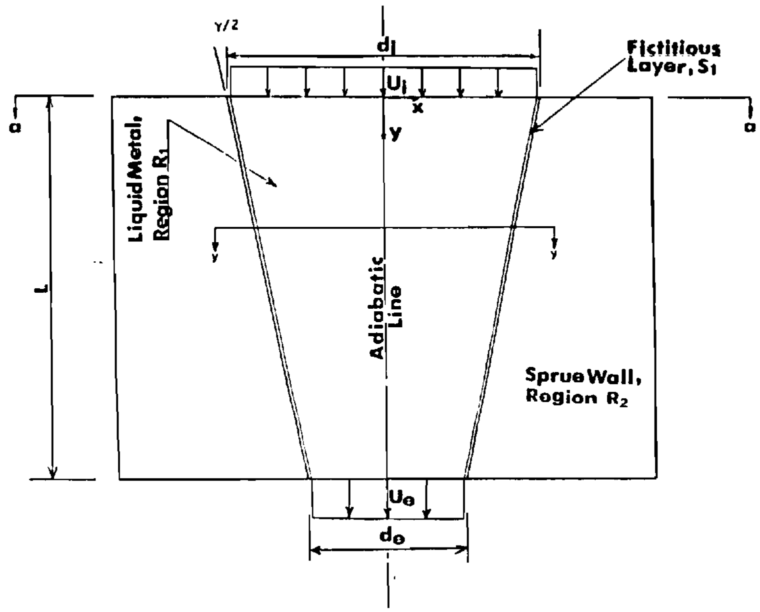

Figure 1. Schematic of a typical gating sprue with slug flow of liquid metal.

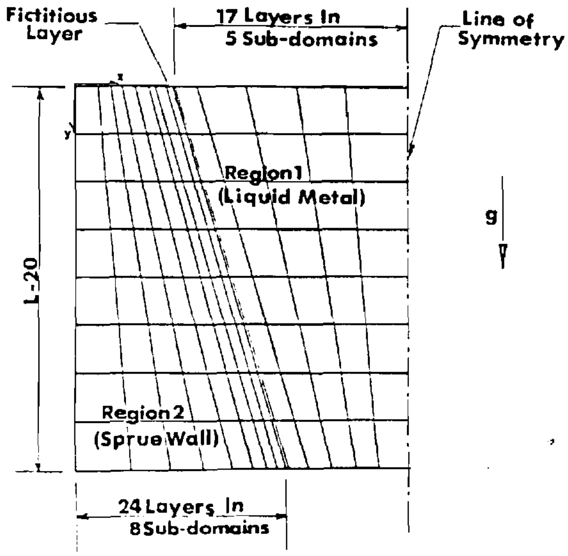

Figure 2. Schematic of the typical finite element mesh.

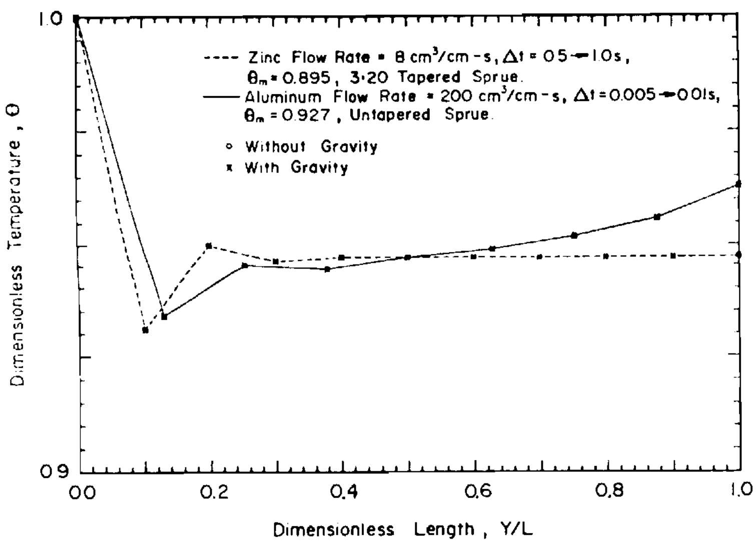

Figure 3. Influence of gravity on the interface temperature after an entering liquid surface has moved 5% of sprue length.

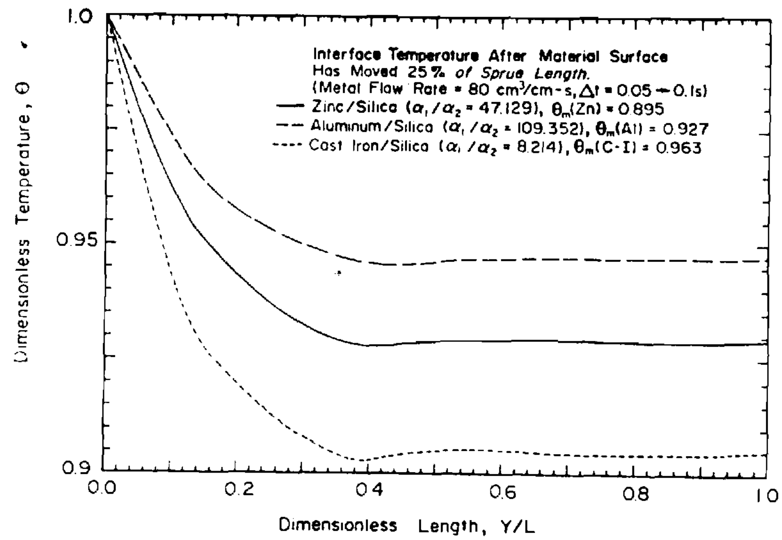

Figure 4a. Influence of thermal diffusivity on the interface temperature for untapered sprues.

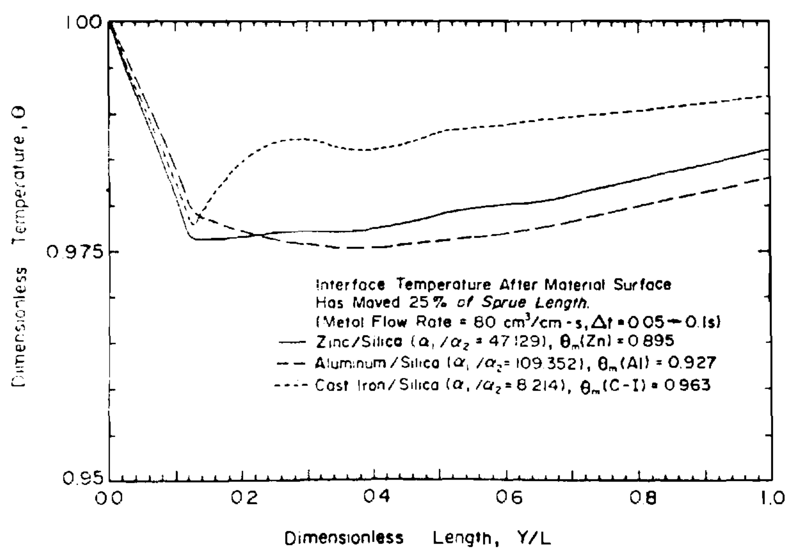

Figure 4b. Influence of thermal diffusivity on the interface temperature for sprues with a 1:10 taper.

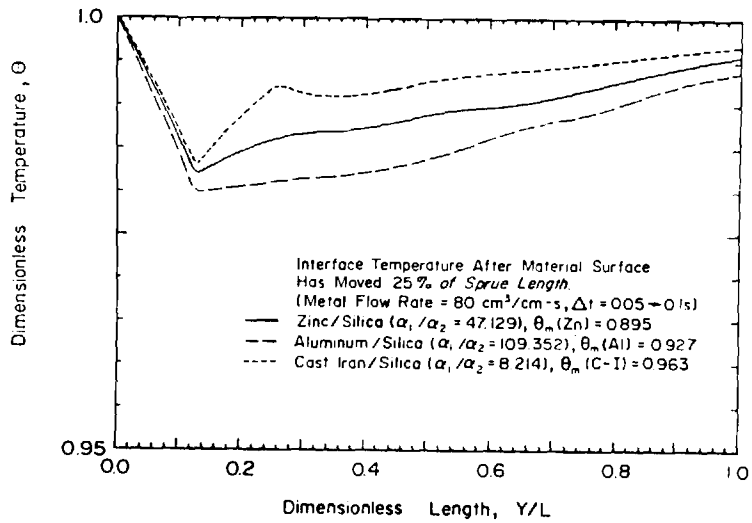

Figure 4c. Influence of thermal diffusivity on the interface temperature for sprues with a 3:20 taper.

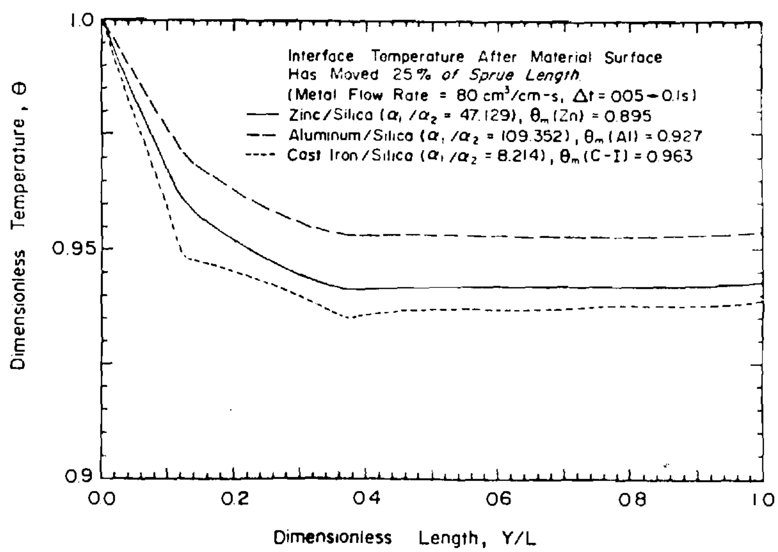

Figure 4d. Influence of thermal diffusivity on the interface temperature for sprues with a 1:50 taper.

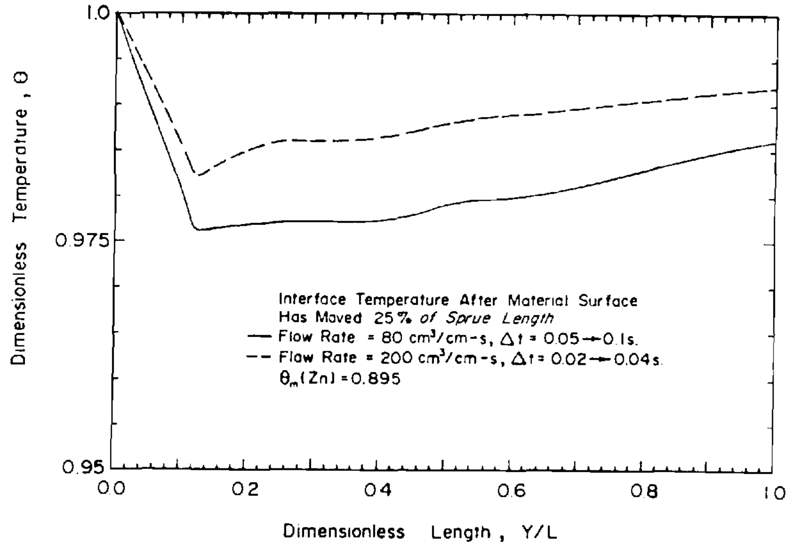

Figure 5. Influence of flow rate on the interface temperature for zinc in silica-sand sprue with a 1:10 taper.

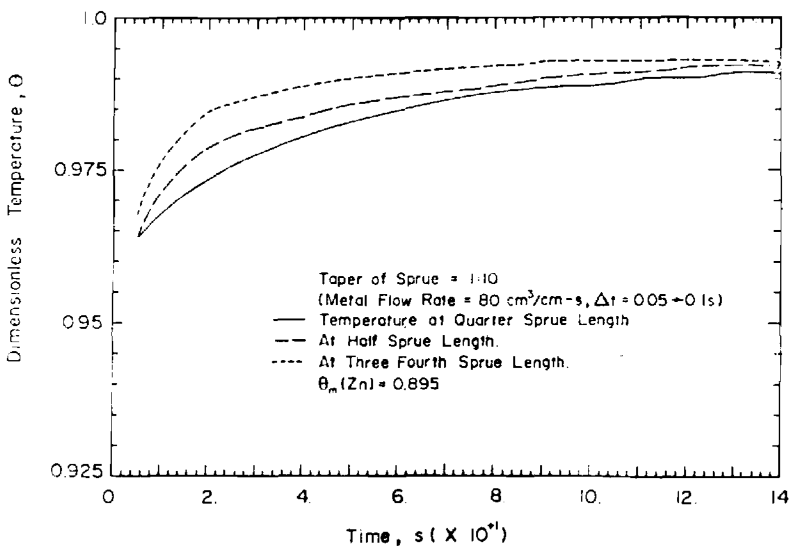

Figure 6. Transient interface temperature at specific locations for zinc in silica-sand sprue.

TASK VII

THE CONTROL AND PRESCRIPTION OF HEAT FLUX AT THE CASTING MOLD INTERFACE

TASK VII

THE CONTROL AND PRESCRIPTION OF HEAT FLUX AT THE CASTING MOLD INTERFACE

The progress on research on this task is summarized in the form of a paper to be presented at the 1986 Winter Annual Meeting of the ASME. The title of the paper is "Some Current Developments in the Computer Modeling of Shaped Casting Solidification". It is written by J. Moosbrugger, a doctoral student in the Woodruff School of Mechanical Engineering at Georgia Tech, J. T. Berry, Professor and Chairman of the Department of Metallurgical Engineering at the University of Alabama, Tuscaloosa, Alabama and C. S. Wei, Assistant Professor in the Department of Mechanical and Aerospace Engineering, Polytechnic Institute of New York, Brooklyn, New York. The entire content of the paper is presented here.

ABSTRACT

Although the use of computers for the simulation of solidification in shaped castings dates back to the nineteen-forties, when Paschkis and co-workers utilized the analogue computer to study this phenomenon, the application in industry has not been widespread. The advent of the digital computer being applied to such problems, which dates from the early nineteen-sixties, focussed the interest of metal casting concerns on the eventual potential of the simulation technique in the production of defect-free products.

Two principal factors have deterred potential users:

i. Non-familiarity with the computation intense software utilized in simulation.

ii. High costs of executing simulations of the solidification of typical industrial castings.

In order to overcome these difficulties several collaborators' efforts have been initiated in various research centers around the world. The authors have been associated with a large-scale project which is being undertaken jointly by three institutions: the Georgia Institute Technology, The University of Michigan and the University of Alabama. IN particular the project concerned looks at problems associated with geometric representation, computational techniques and data base exploration. The present paper examines developments in the field of solidification simulation in shaped casting and in particular discusses some alternative approaches to minimizing cost in such simulations.

Special emphasis is placed on techniques for reducing these costs which involve applying special boundary conditions at the mold-metal interface in order to eliminate nodal points in the mold, locations which are only the secondary interest to foundry methods engineers.

Amongst the approaches which are discussed are the following:

a. Analytical Methods

b. Numerical Solutions Utilizing Similarity

c. Boundary Curvature Method

d. Boundary Element Method

e. Numerical and Experimental Studies

In each case the validation of the approach concerned using thermal analysis of shaped castings is emphasized.

In the paper it is concluded that possible combinations of the alternative approaches may be worth further exploration in view of other individual limitations. It is envisaged that the generation of heat flux data maps will probably result from this combined approach, essentially reducing the description at the metal-mold interface during solidification to a boundary condition of the second kind. Finally the possibility of control of interfacial heat flux during solidification of shaped castings is discussed in the light of preliminary modeling in this area.

INTRODUCTION

The emergence of the digital computer in the last few decades as an economical tool has made it possible for the engineer to find accurate numerical solutions to boundary value/initial value problems for complex geometries and non-linear formulations. Thus, it is no surprise to find a great deal of activity in the recent literature in the area of solidification heat transfer. The supply of high quality sand castings is vital to many manufacturing industries. Thus, any information leading to improvements in productivity and/or efficiency of the casting process or to improvements in the quality of the product is of considerable interest. Mathematical models of the solidification process can provide such information. Ideally, the optimum placement of risers, chills and insulating mold materials in the rigging for a sand casting can be determined via computer simulation of the solidification process. Thus, the foundryman is provided with the information needed to achieve the level of soundness and/or mechanical properties in designated locations of the casting. This information has, in the past, been largely obtained empirically through a trial and error process.

Numerical solutions for sand casting solidification problems, unfortunately, are limited in their general applicability to industrially important problems because of the relatively large storage and central processor time requirements encountered in modeling the complex geometries inherent in sand cast parts. Coupled with the transient nature of the problem and the non-linearities encountered, the inherent complexity of sand cast parts results in extremely computationally intensive numerical solutions, ideally, involving sets of governing equations describing a coupled heat, mass and momentum transport problem. The state of the art in this area, however, involves solutions only to uncoupled heat conduction equations which include latent heat generation.

The subject dealt with in this paper relates to the aforementioned limitation of computational cost on two distinct levels. First, it may be possible to significantly reduce the computer capacity required by replacing the discretized solution space of the mold material in finite element (FEM) or finite difference (FDM) models of sand casting problems with equivalent heat transfer coefficients applied at the mold-metal interface. Secondly, in order to arrive at an initial design for the gating and feeding of a given casting the engineer may make use of so called "first level" software. That is, software based on less rigorous mathematical models and empirical information allowing a first estimate of the optimum design. Inevitably the accuracy of the "first level" will determine the number of iterations required in the second level which is, ideally, the numerical solution to a set of momentum, mass, and energy conservation equations with an appropriate set of boundary and initial conditions.

On both levels, the accurate description of the heat flux and/or integrated rate of heat transfer at the mold/metal interface is important. In the first case, heat transfer coefficients applied as boundary conditions at the mold/metal interface must accurately model the effect, both locally and globally, of the mold material they replace. In the second case, accurate calculations based upon freezing order principles (i.e. modulus based riser design based on variants of Chvorinov's rule) rely upon geometric correction factors which account for diverging and converging heat flux at corners and curved surfaces.

In this paper, the recent literature will be reviewed with emphasis upon both analytical, and approximate models for describing the rate of heat transfer at the mold-metal interface., Recently proposed techniques for representing the effects of sand molds with equivalent heat transfer coefficients applied at the casting-mold boundary will be discussed.

HISTORICAL PERSPECTIVE

A great deal has been written in the recent literature relating to computer simulation of sand casting solidification. It is not intended to give an exhaustive survey here. However, a brief, historical introduction will be pursued in the light of the subject at hand.

The general problems of melting and freezing are of interest in many practical and current engineering applications. A common feature associated with such problems is the occurrence of a moving phase change boundary. Energy conservation applied at this boundary yields a non-linearity in the mathematical model which can be further enhanced if temperature dependent thermal properties are considered. The first mathematically exact solution of a moving boundary problem was discussed by Neumann in his lectures in the 1860's [1] though his notes were not published until 1912 [2]. Stefan [3] published analytic work on the formation of polar ice in 1891, and his name is often associated with moving boundary problems, though his formulation was actually a special case of the problem discussed by Neumann in that the initially liquid region was assumed to be at the fusion temperature. Neumann's solution was extended by Schwarz [4,5] to a one-dimensional system consisting of a solidifying liquid in contact with a heat absorbing solid. Schwarz gave mathematically exact solutions to the governing temperature field equations in the heat absorbing solid, the solidified region and the liquid region all of which were assumed to have constant, though unequal thermal properties. The problem considered by Neumann [1] dealt with a solidifying liquid which had the boundary maintained at a constant temperature lower than the solidification temperature. An important result of Schwarz's solution is that the temperature of the interface between the heat absorbing solid and the solidified liquid is constant for all times $(t > 0)$. . Following Schwarz's analysis, Chvorinov [6] developed his rule for solidification times of simple shaped castings which is now famous amongst foundrymen.

For two-dimensional moving boundary problems, no exact mathematical solutions exist though approximate analytical techniques have been used to provide solutions for such problems [7.8.9.10.11]. Analytical solutions to three-dimensional problems of this type are virtually infeasible. Thus, for practical problems the engineer must turn to numerical techniques which hold much promise.

Several publications have reviewed the literature specifically pertaining to sand casting solidification simulation. A bibliography of research literature on this subject was given by Durham and Berry [12] in 1974. More recently, Erickson [13] has reviewed the literature providing a long list of citations. Recent work describing the use of FEM and FDM models in sand casting solidification problems can be found in References 14 and 15. Most recently, Thomas, Samerasekera and Brimacombe [16] have published the results of a comparison of numerical modeling techniques for ingot solidification problems. The aforementioned publications have focused primarily upon FEM and FDM algorithms and the techniques used to represent latent heat release, time differencing schemes and their relative merits and demerits.

Recent attention has been given in the literature to the problem of describing the time and position dependent heat flux at the casting/mold interface in sand casting solidification problems. Should the accurate description of this heat flux be available, significant reductions in the amount of computer time required can be achieved because the majority of the nodal points in an FEM or FDM model occur in the mold region. These could be eliminated by applying equivalent, time dependent, boundary conditions, which represent the heat absorbing effect of the mold, at the casting-mold interface. In the following sections, recently proposed formulations and implementations of these boundary conditions will be discussed.

ALTERNATIVE APPROACHES TO REDUCING COMPUTATIONAL COSTS IN SOLIDIFICATION SIMULATION

Several alternative approaches have been suggested which ostensibly lead to reduced computational costs in the solidification simulation of shaped casting. Among these are several methods involving the compact representation of interfacial heat flux at the mold-metal boundary. The boundary element method (BEM) has also attracted recent attention. The following sections discuss each of these approaches, in turn their potential advantages, as well as disadvantages. Several aspects of unpublished work or work in progress are referred to in each individual discussions.

Analytical Solutions

The first reported use of an analytical solution for interfacial heat flux in replacing the nodal points of the mold in a sand casting problem can be attributed to Niyama [17]. Niyama used the classical result that, for a one-dimensional semi-infinite body with constant thermal properties whose surface ($x = 0$) is maintained at a constant temperature (T_i) and which is initially at a uniform temperature (T_o) the heat flux at the surface is given by

$$q = [K/(\pi\alpha t)^{1/2}](T_i - T_o) \qquad (1)$$

Here K is the thermal conductivity, α is the thermal diffusivity and t is time. Niyama derived finite difference equations using simple energy balances at nodal points. He compared results for two-dimensional steel casting simulations with those using the usual FDM models with a fully discretized mold. Some discrepancies were found between the two techniques and were attributed to the neglect of mold cavity shape on the heat flux function (equation 1).

A natural extension of the technique discussed by Niyama is to seek mathematical descriptions for the heat flux present at interfaces with two- and three-dimensional characters. Rastegar and Desai [18] applied experimentally determined heat flux functions for a two-dimensional rectangular mold cavity in a study of natural convection effects. Later Berry [19] suggested the use of a heat flux map for replacing the chilling effects of the mold. Subsequently, Wei, Hansen and Berry [20] published the results of an initial effort at describing the heat flux present at the mold-metal interface for two-dimensional right angle corners and discussed compact descriptions for the heat flux at cylindrical and spherical interfaces.

Approximate analytical expressions for the cylindrical and spherical interfaces are, respectively

$$q = (t_i - T_o)(K/(\pi\alpha)^{1/2})[1/t^{1/2} + (\pi\alpha)^{1/2}/2r_o] \qquad (2)$$

$$q = (T_i - T_o)(K/(\pi\alpha)^{1/2})[1/t^{1/2} + 1/r_o] \qquad (3)$$

where r_0 is the radius of the cylinder or sphere. Equations (2) and (3) represent approximate solutions (valid for small values of time, t) for the heat flux at r_0 for semi-infinite regions bounded internally by a circular cylinder or a sphere where the boundary ($r = r_0$) is held at a constant temperature, T_i, and the region is initially at a uniform temperature, T_0 [21]. Wei, et al [20], further, discuss the use of product solutions in order to formulate the heat flux present for two dimensional corners. Preliminary results of a comparison of finite difference solutions for a problem with two-dimensional symmetry, one employing mesh points in the mold region and the other employing analytical solutions for the interfacial heat flux, were reported. These results indicated that the technique could provide good results.

Wei [22], was able to formulate a general analytical expression for the heat flux occurring at corners of arbitrary angles. He studied the problem governed by the following equation and boundary conditions:

$$(1/r)(\partial/\partial r)(r\partial\phi/\partial r) + (1/r^2)(\partial\phi/\partial\theta^2) = (1/\alpha)(\partial\phi/\partial t) \qquad (4)$$

with

$$\begin{aligned} \phi(r,\theta,0) &= 0 \\ \phi(r,0,t) &= 1 \\ \phi(r,\theta_0,t) &= 1 \end{aligned} \qquad (5)$$

where

$$\phi = (T - T_0)/(T_i - T_0) \qquad (6)$$

A semi-infinite region bounded by a wedge of arbitrary angle is shown in Figure 1,

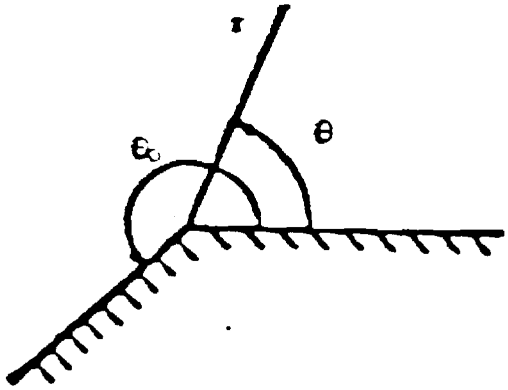

Figure 1. Coordinate system for wedge or corner configuration ($0 < \theta_o < 2\pi$).

The solution to (4) and (5) is given by Jaeger [23] as

$$\phi = 1 - (4/\theta_0) \sum_{n=0}^{\infty} \sin[(2n + 1)\pi\theta/\theta_0] \int_0^{\infty} e(-u^2F_0)\ (J\nu(u)du)/u \quad (7)$$

where Fo is the Fourier number $\alpha t/r^2$, J_ν is the Bessel function of the first kind of order ν and $\nu = (2n + 1)\pi/\theta_0$ for n \ 0,1,2,... The heat flux over the wedge enclosure can then be derived as

$$q(r,t)=(-K/r)(\partial\phi/\partial\theta)\Big|_{\theta=0} = (4K/r\theta_o)\sum_{n=0}^{\infty}[(2n+1)\pi/\theta_o \int_0^{\infty} e(-u^2F_o)(J_{\nu}(u)du)/u \quad (8)$$

Wei defines a dimensionless edge function as the ratio of the heat flux for a two-dimensional corner to the heat flux for a one-dimensional interface, that is

$$E_{\theta o}(F_o)=[q(r,t)]/[q_{1d}(t)]=4(\pi F_o)^{\frac{1}{2}}\sum_{n=0}^{\infty}[(2n+1)\pi/\theta_o{}^2] \int_0^{\infty} e^{(-u^2F_o)}(J_{\nu}(u)du/u) \quad (9)$$

The above expression is governed by the wedge angle, θ_0, and the Fourier Number, Fo, and, thus, embodies a similarity in distance from the corner, r (with $\theta = 0$ or $\theta = \theta_0$), and time, t. The above expression can be evaluated [22] and is shown graphically in Figure 2 for various wedge angles, θ_0. For a region which is bounded internally by a composite of wedge angles as shown in Figure 3, one cannot obtain a closed form solution for the interfacial heat flux. Wei [22], introduced two approximate forms for the interfacial heat flux at such an isothermal cavity wall. These approximate forms differed according to the assumed form of corner interaction and were termed additive and multiplicative. With reference to Figure 3, these were given as

$$q_{add} = q_{1d}[E_{\theta 1}(r_1/2(\alpha t)^{1/2}) + E_{\theta 2}(r_2/2(\alpha t)^{1/2}) - 1] \qquad (10)$$

$$q_{mult} = q_{1d}[E_{\theta 1}(r_1/2(\alpha t)^{1/2})E_{\theta 2}(r_2/2(\alpha t)^{1/2})] \quad (11)$$

Wei [22] compared these two formulations by testing their performance in a problem with a known analytical solution. A semi-infinite, rectangular enclosure as shown in Figure 4 was chosen and the results are shown in Figure 5. Choosing the additive form (equation 10), a heat transfer coefficient, h, defined by

$$h(T_i - T_o) = -K_c(\partial T/\partial n) - K(\partial T/\partial n) \quad (12)$$

can be defined for a metal casting problem, where K_c is the thermal conductivity of the cast metal and n is directed along the outward normal to the surface. Thus,

$$h = (K/(\pi\alpha t)^{1/2})[E_{\theta 1}/(\pi\alpha t)^{1/2}) + E_{\theta 2}(r_2/\pi\alpha t)^{1/2}) - 1] \quad (13)$$

referring, again, to Figure 3. Wei compared the results of FDM solutions for a right angle corner region using properties for aluminum cast in dry silica sand. He compared results for (a) conventional FDM using temperature dependent thermal properties, (b) conventional FDM using constant thermal properties and (c) FDM using the above formulation for interfacial heat flux. His conclusion, based upon solidification front position at various times, was that method (c) did not result in any significant loss of accuracy while significantly reducing computation time. His results are shown in Figure 6.

Use of Numerical Solutions Utilizing Similarity

In many problems governed by partial differential equations, it is possible to reduce the number of independent variables appearing in the governing equations via a similarity transformation. That is two or more variables are combined into a single variable and the governing equations and boundary conditions are re-written in terms of the new variables. Zheng and Wei [24] have published the results of a similarity analysis of heat conduction in a wedge enclosure with or without change of phase. For a wedge enclosure subject to a uniform and constant boundary temperature, they were able to show that by integrating the local heat flux from r = 0 (the wedge apex) to an arbitrary r = r' along the boundary surface, a similarity in position, r, along the wedge and time, t, can be obtained. They denote this integrated heat flux by $Q_\Lambda{}^c$ and show that it can be written in terms of the thermal conductivity and a geometric factor. That is

$$Q_c{}^\Lambda = KG^c(\Lambda,\theta_o)[T_i - T_o] \tag{14}$$

where G^c is given by

$$G^c(\Lambda,\theta_o) = (4\pi/\theta_o{}^2) \sum_{n=0}^{\infty}(2n + 1) \int_0^\Lambda\int_0^\infty e^{-(u/2r^*)^2} (J_\nu(u)du/u)(dr^*/r^*) \tag{15}$$

and can be regarded as a geometric factor since it depends only on the wedge angle, θ_o, and the similarity parameter Λ where un-numbered equation Λ

$$\Lambda = (r'/(4\alpha t)^{1/2})$$

For the case of heat conduction in a wedge with change of phase, a similarity in the integrated heat flux was also found. They consider the following moving heat source problem

$$(\partial^2 T/\partial r^2) + (1/r)(\partial T/\partial r) + (1/r^2)(\partial^2 T/\partial \theta^2) = (1/\alpha)(\partial T/\partial t) \qquad (16)$$

with

$$\begin{aligned} T(r,0,t) &= T_i \\ T(r,\theta_o,t) &= T_i \\ T(r,\theta',0) &= T_o \\ T(\infty,\theta,t) &= T_o \end{aligned} \qquad (17)$$

and

$$T = T_1 = T_2 = T_F,\ 0 < \theta < \theta_o,\quad r = R(\theta,t),\ t > 0$$

with

$$(K_1(\partial T_1/\partial r) - K_2(\partial T_2/\partial r))[1 + (1/R^2)(\partial R/\partial \theta)^2] = \rho L(\partial R/\partial t) \qquad (18)$$

at the moving phase change boundary. Using the solution of Budhia and Kreith [11], who separate the problem into a pure heat conduction problem and a moving boundary problem, they are able to show that the integrated heat transfer rate can be written as

$$Q_\Lambda = -K_1 G^c (T_F - T_i)(1 + T_o^*) - K_1 G^m (T_F - T_i) \qquad (19)$$

where

$$T_o^* = (K_2(T_2 - T_F)/K_1(T_F - T_i)) \qquad (20)$$

and

$$G^m = (4\beta/\theta_o^2) \sum_{n=0}^{\infty} n\pi \int_0^{\Lambda} \int_0^{1} (d\tau/(1 - \tau) \int_0^{(\theta_2/2)} \eta^2 \exp[(-n^2 \tau + r^{*2})/(1-)]$$

$$\times I'_p [(2\eta(\tau)^{1/2})r^*)/(1 -)] \times (\sin(n\pi\theta'/\pm J_0)d\theta'(dr^*/r^*))] \qquad (21)$$

is a geometric factor for the moving source problem. I_p' is a modified Bessel function of the first kind, $p' = (n\pi/\theta_0).G^m$ depends on the parameters θ_0 and Λ and also on β, the ratio of latent heat to sensible heat and η, the dimensionless interface position which is a function of β, λ and T_0^*. The constant λ can be determined and is equal to the interface constant in a one-dimensional Stefan problem. The geometric factors, G^c and G^m can be evaluated numerically and are given by Zheng and Wei [24] for a range of Λ.

For all of the aforementioned results, a constant interface or boundary temperature was assumed apriori. Zheng [25] has also investigated the case of a two-dimensional solidification system. That is, he studied the governing equations and boundary conditions for the case of a solidifying liquid region in contact with a heat absorbing solid with a wedge geometry. For the case of constant but unequal properties in the solid, solidified layer and liquid regions, he was able to reduce the number of variables in the governing equations and boundary conditions and the conditions at the moving interface to two, the angular variable θ and the dimensionless $r^* = r/(4\alpha t)^{1/2}$. Though the problem cannot be solved analytically, the similarity, once established, can be used to reduce the number of independent variables needed for data reduction. Zheng verified the existence of this similarity with a conventional finite difference solution.

An interesting result of Zheng's numerical solutions was that the integrated heat flow rates $Q_\Lambda{}^s$ for the two-dimensional solidification systems studied always exceeded the integrated heat flow rate Q_Λ for the pure heat conduction problem with constant boundary temperatures equal to the interface temperatures of one-dimensional solidification systems.

Zheng [25] has also studied the case of three-dimensional heat conduction with or without phase change. He did not report an analysis of a three-dimensional solidification system however. He was able to establish similarity for both cases and expressed his results (obtained numerically) as an integrated edge function

$$E = (Q_{3d})/Q_{1d} \tag{22}$$

The integrated edge function, E, is a function of a dimensionless edge coordinate Λ and a dihedral angle θ_{xy}. Zheng has tabulated results for several cases [25].

As the principal of similarity can be used to reduce the number of independent variables in a problem, it represents a technique for reducing storage requirements for analytically or numerically determined boundary heat flux functions. Though similarity cannot be found for local heat flux, the integrated heat transfer rate can be used to determine local integrated heat transfer rates over discrete boundary areas and therefore to find average heat fluxes over, for example, the boundary surface of a finite element.

Solutions for problems involving pure heat conduction with phase change might be used to approximate the case of a sand mold with a volatile binder (i.e. green sand molds). However, these cases actually constitute a coupled heat and mass transfer problem with secondary effects such as moisture recondensation. [26].

Boundary Curvature Method

Dantzig and Lu [27] have described a systematic technique for estimating the local heat flux in sand casting solidification and have described its implementation in the FEM code [28]. Termed the "boundary curvature method", this technique represents the most advanced method yet reported in terms of practical applicability for two distinct reasons. First, it is highly general and can be applied to almost any shape. Secondly, as described, the method uses solutions which (at least approximately) account for the effect of moisture in green sand molds and, thus, can be used to model the solidification of parts cast in green sand.

The technique consists of assigning a local curvature to each point on the surface of a casting and applying the heat flux which would be present for a cylinder or sphere of equal curvature. The idea here is that heat flows normal to isotherms and that the isotherms surrounding a corner or edge resemble, locally, those surrounding a cylinder or sphere. Thus, the local extent of divergence or convergence of heat flux is taken into account by assigning an equivalent curvature to each point on the surface.

In order to determine the appropriate heat flux to be used at each point on the surface of a casting with this technique it is necessary to determine the appropriate two or three-dimensional equivalent curvature. Dantzig and Lu do this in two-dimensional parts, for example, by finding two points on the surface of the casting which are at a distance, ℓ, from the point of application given by

$$\ell = (\alpha t)^{1/2} \tag{23}$$

A circle may be fitted through three points on the surface and its radius computed. The heat flux occurring at the surface of a cylinder of equal radius for the same time, t, can then be applied as a boundary condition at that point. The technique can be extended to three dimensional geometries by using combinations of cylinders and spheres (and planes) and, therefore, is highly general. However, it should be pointed out that the method for determining equivalent curvature at a point is not based upon rigorous analysis. It is essentially a notional approach based upon a dimensional argument. As such, it may embody limitations which can be borne out only by experimentation or comparison with conventional FEM or FDM solutions for a wide range of geometries.

In determining the time dependent heat flux for planar, cylindrical and spherical interfaces. Dantzig and Lu use FDM solutions for the appropriate one-dimensional heat conduction problems with phase change. By treating the heat transport in the mold as a phase change or moving heat source problem, the effect of the binder in bonded sand molds is accounted for, at least to the extent that the latent heat of vaporization of the volatile component is being consumed at a moving boundary. In addition, numerical solutions allow the thermal properties to vary with temperature. While it should be pointed out that the problem of heat transport in green sand molds is, in reality, a more complicated matter involving coupled heat and mass transport in a porous medium [25] and can involve such effects as recondensation of moisture as pointed out by Dantzig and Lu, the incorporation of change of phase into the solutions represents a more realistic description of the heat transport for such applications than do the techniques discussed earlier.

The application of these numerical solutions, once generated, is accomplished by referencing a library of computed heat fluxes during the numerical solution to the casting solidification problem. Once stored as a function of radius, boundary temperature, and time they can be used to provide an effective heat flux or heat transfer coefficient at each point on the casting surface once the equivalent curvature has been determined.

Dantzig and Lu performed a comparison between the boundary curvature method, a conventional FDM solution and experimental results. In light of the highly general nature of their technique and the uncertainty inherent in the material properties used, the results reported showed good agreement and indicated that the technique bears promise.

Boundary Element Method

Hong, Umeda and Kimura, [29,30] have discussed the application of a relatively new numerical method, the boundary element method (BEM), to solidification problems. They presented a formulation for a coupling model in which the solidification problem in the casting is solved using an explicit finite difference method and the heat conduction problem in the mold is solved using BEM. Later, they discuss a formulation wherein the problems in both the casting and mold are solved using the BEM. Time marching schemes for obtaining accurate solutions and data for the proper range of time-steps are presented. As a validation of their models, they present a comparison between the coupling model and a conventional finite difference solution to a two-dimensional solidification system and a comparison of the BEM and a conventional finite difference solution. The results showed excellent agreement between both models and conventional FDM.

The attractiveness of the boundary element method lies in the fact that the system of equations to be solved at each time step involves only unknowns at the boundary points of the solution domain. While the entire solution domain must be discretized in order to evaluate integrals which occur in the system matrices, the system of equations which results from application of the BEM involves only the dependent variables at the boundary nodes. Thus, at each time step in the numerical solution, the number of simultaneous equations to be solved is reduced by one dimension from the order of the problem. This results in considerable savings in computational effort as compared to FEM and FDM for complex to and three-dimensional problems.

A possible drawback to the BEM is the fact that a fundamental solution to the governing equation for heat conduction with variable thermal properties;

$$\rho c(\partial T/\partial t) = \nabla \bullet K \nabla T \qquad (24)$$

does not exist. Such a fundamental solution is a prerequisite for a BEM formulation of such a problem. In contrast FDM and FEM formulations can be obtained for this class of problems.

Thus, average properties must be used. However, as pointed out by Hong et. al. [20], a modified temperature scale can be defined as

$$\phi = \int_{T_{ref}}^{T} K/K_{ref}\, dT \tag{25}$$

so that equation (24) becomes

$$\partial\phi/\partial_t = \alpha\nabla^2\phi \tag{26}$$

where $\alpha = k/\rho c$ is a function of ϕ. T_{ref} is a reference temperature and K_{ref} is the thermal conductivity at that temperature. If α is not as strongly dependent on temperature as K so that it may be reasonably assumed to be constant, a fundamental solution can be found for equation (23) and, thus, a BEM formulation can be developed.

It may also be noted that, should one wish to approximate the thermal transport problem in green sand molds with a formulation analogous to the case of melting of a congruently melting material, the temperature recovery algorithm incorporated in a BEM formulation as presented by Hong et. al. [29,30] would be suitable.

Numerical and Experimental Studies

The effects of corners upon the heat extractive capacity of the mold cavity has been appreciated by practitioners for many years. An elegant experimental study of Ruddle and Skinner [31] performed in the early nineteen fifties documented these effects quite comprehensively and provided tabulated data which could be utilized to correct calculations of solidification time based upon the Chvorinov rule. Later Berry et alia [32] indicated how such data could be effectively applied to refine the Chvorinov rule [6] relating solidification time to the square of the casting volume to surface area ratio. This ratio, frequently termed that 'casting modulus' by foundrymen, was modified to incorporate the effective surface area of the casting. This area was calculated using the corner factor data tabulated by Ruddle and Skinner [31].

Berry and co-workers [32] provided experimental verification of this approach and further showed how this modification to the casting modulus extended the effectiveness of the original Chvorinov relationship to heavier sectioned castings where corner effects were known to be significant.

More recently Franklin [33] has utilized the approach of Ruddle and Skinner, which involved the experimentally determined isotherms occurring around corners of various types (See Figure 7). Wei, Berry and Franklin [34] have briefly described the manner in which this technique was extended to examine the interfacial heat flux variation during the solidification of a high purity aluminum casting containing a variety of two and three-dimensional corner configurations. The results are presented in the form of curves relating the dimensionless heat flux to dimensionless time (Figure 8). In the figure concerned the results of experiments are compared with those of Jaeger's analytical solution [23]. The experimental results were seen to bracket the appropriate analytical solution, where such solutions were available. An interesting observation made by Franklin [34] in reviewing the temperature gradient data obtained in the course of his experiments, was that there were two sources of divergence from the linear relationship predicted by Chvorinov [6]. The two sources appeared to be related to both molten metal super heat and to corner geometry. The analytical method of Jaeger cannot, of course, account for superheat, consequently this source of divergence is to be expected. However, the fact that the geometric effect appears late in time is of great interest. This suggests that for thinner sectioned castings (<10 mm) corner effects will be much less dominant than those of thicker section (> 50 mm) (See Figure 8).

The extension of the technique developed by Franklin to more complex configurations, for example to combinations of geometries such as a cruciform, necessitates using a numerical method whereby the temperature gradient term in the expression for interfacial heat flux is obtained by an FDM or FEM based scheme. Moosbrugger [35] has used such a method to determine the Edge Function for various configurations. A finite element model was utilized to solve the field equations in the solidifying metal and in the mold for one and two dimensional geometries, allowing relaxation of the idealized conditions assumed in analytical solutions appropriate to the various simple cases. Mold-metal boundary heat fluxes were then computed and validations of the method undertaken for geometries where analytical solutions exist. Figure 9 shows an Edge Function curve for a portion of the cruciform geometry, so determined. The case simulated again involved high purity aluminum in silica sand molds. For the combinations of metal and mold material studied, it appears that available analytical forms for the interfacial heat flux can be used to describe that quantity if appropriate average thermal properties are chosen for that combination, and providing superheat is neglected. If thermal properties which are temperature dependent are used in the simulation and if superheat it introduced, one must rely entirely on the predictions of the simulation in building data on the Edge Function. None the less, the cost and possible scope of such computational experiments offer a greater flexibility than does the purely experimental approach to mapping interfacial heat flux data.

CONCLUDING DISCUSSION

The usefulness of solidification simulation as a tool in the metal casting industry is now well established. However, its use is not yet widespread because of either a lack of familiarity of potential users with the computationally intense software or because of the high costs of executing such simulations. This paper has described how a variety of techniques can be pressed into use to minimize such costs and at the same time render into a more compact form, the input required in such simulations. Instead of burdening both the investigator and the computation system with determining the thermal history of the surrounding mold, which will often contain upward of seventy percent of the elements involved in an FEM simulation, it appears that several techniques may be used, either singly or in combination, to achieve this end. It seems highly likely that combinations of heat flux data in the form of maps appropriate to various mold geometries and obtained using both numerical and analytical methods will be the principal adjunct involved. It would appear that both analytical and numerical methods will have their place in building these heat flux maps, analytical techniques being appropriate where thinner casting sections, simple geometries and low casting superheats with short-freezing range alloys are involved. Numerical approaches would be employed where more complex, thicker section castings involving long freezing range alloys are used.

Since many commercial and premium quality cast products demand guaranteed mechanical properties, it would clearly be desirable to begin the implementation of the above approach on a limited scale using instrumented mold assemblies. At the same time, as knowledge develops regarding the relationship between casting thermal history and microstructural evolution (and hence with mechanical properties), it would also seem to be appropriate to consider the provision of predetermined heat flux patterns to the casting. The prescription of the heat flux pattern will encompass much work of a multidisciplinary nature. However it would appear prudent to explore the possibility of using the heat-pipe device [36] to maintain the interfacial heat flux patterns desired in critical regions of the casting, not only during the solidification but possibly also throughout solid state transformations that occur in cooling and which subsequently affect its ultimate performance.

ACKNOWLEDGEMENTS

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NOMENCLATURE

C	specific heat
E	integrated edge function
$E_{\theta o}$	dimensionless edge function
Fo	Fourier number, $\alpha\tau/r^2$
G^c	geometric factor for pure conduction problem
G^m	geometric factor for moving source problem
h	heat transfer coefficient
K	thermal conductivity
L	latent heat of fusion
Q_c^Λ	integrated heat flux for wedge
Q_Λ	integrated heat transfer rate for wedge with phase change
Q_s^Λ	integrated heat flux for 2-D solidification systems
q	heat flux
R	position of phase change interface
r	radial coordinate
r'	arbitrary distance along wedge
r_o	radius of cylinder or sphere

T	temperature
T_i	boundary temperature
T_o	initial temperature
T_F	melting or fusion temperature
t	time
α	thermal diffusivity
β	ratio of sensible heat to latent heat
Λ	similarity parameter along edge, $r'/(4\alpha t)^{1/2}$
λ	interface constant in 1-D Stefan problem
η	dimensionless interface position
ϕ	dimensionless temperature, modified temperature in equations (25) and (26)
ρ	density
θ	angular coordinate
θ_o	wedge angle
∇	divergence operator
∇^2	Laplacian operator

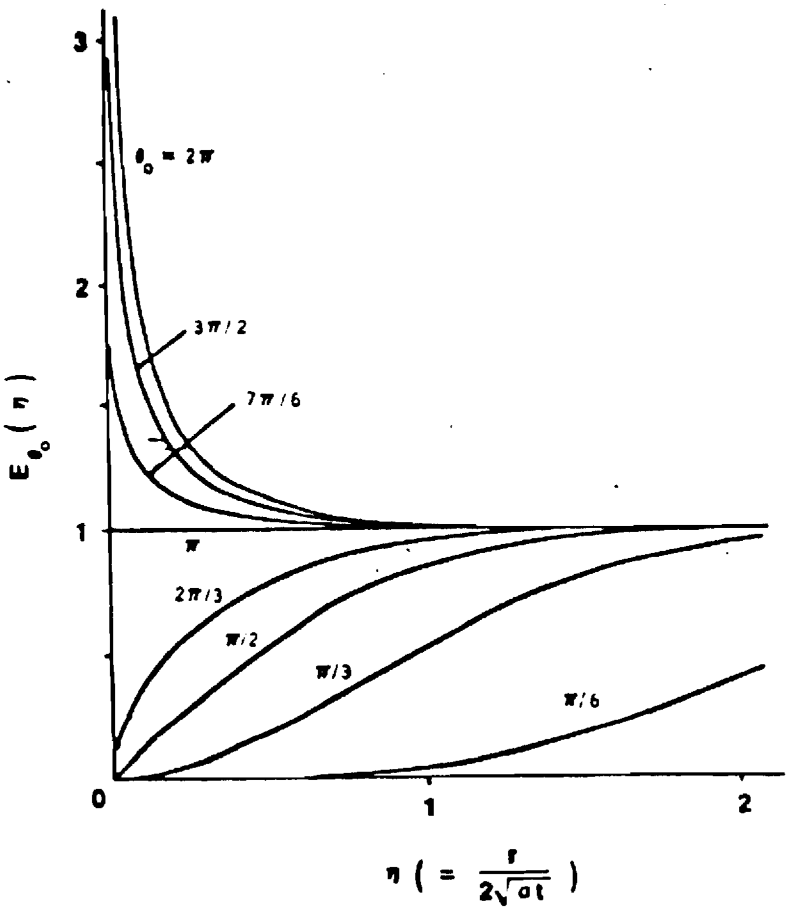

Figure 2. Graphical representation of the E-function, vs. $r/2(\alpha t)^{1/2}$.

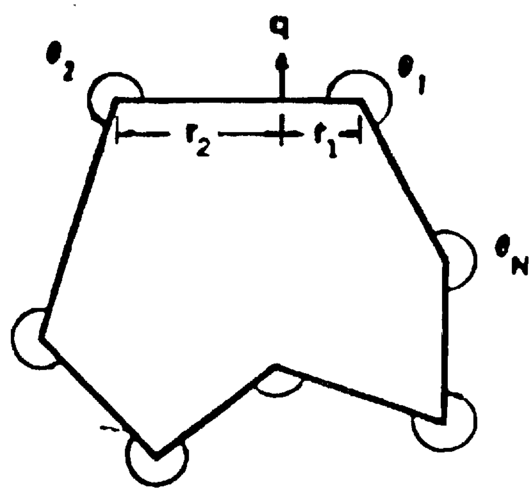

Figure 3. Cross-section of an infinite solid having a cavity of the shape of a polygonal cylinder.

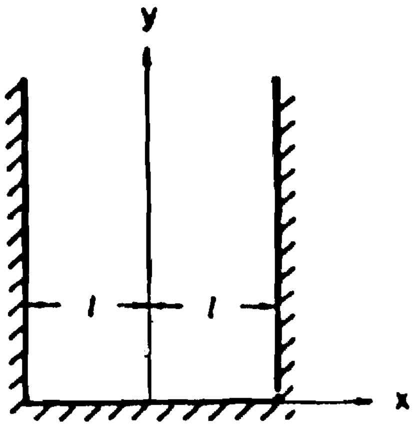

Figure 4. Semi-infinite rectangular enclosure.

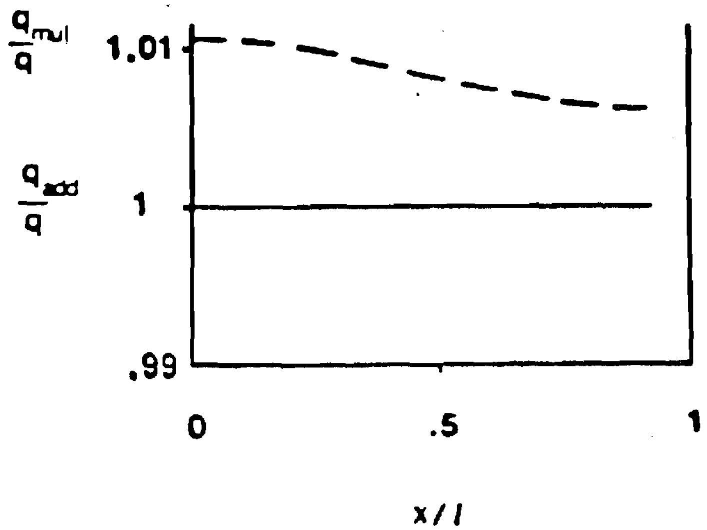

Figure 5. Comparison of approximate q-functions, with the analytical solution, $\alpha\ t/\ell^2 = 0.18$.

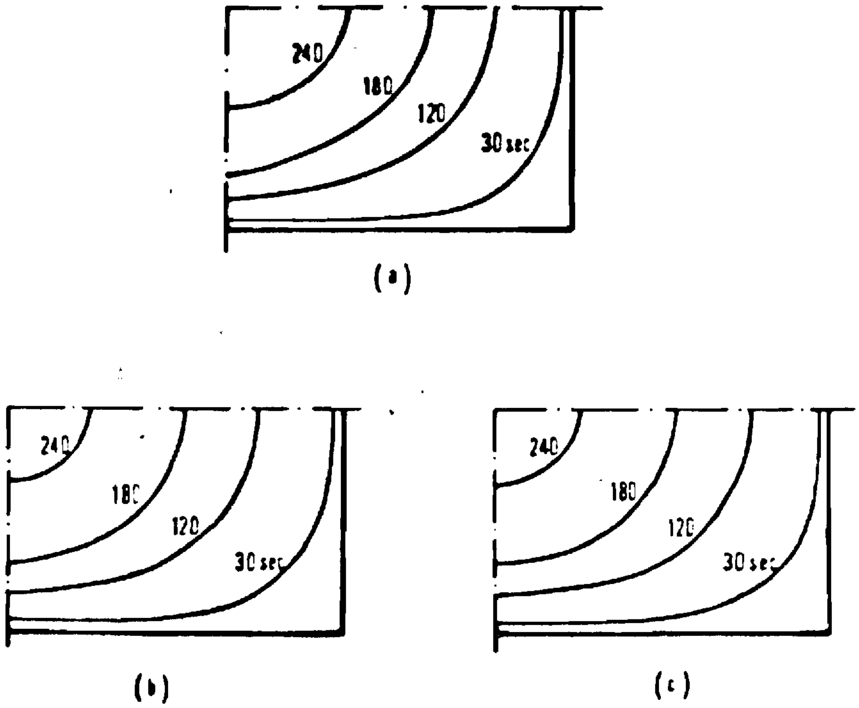

Figure 6. Computed solidification front movements via (a) conventional approach incorporating the temperature dependence of mold properties, (b) conventional approach using constant mold properties, and (c) the q-method.

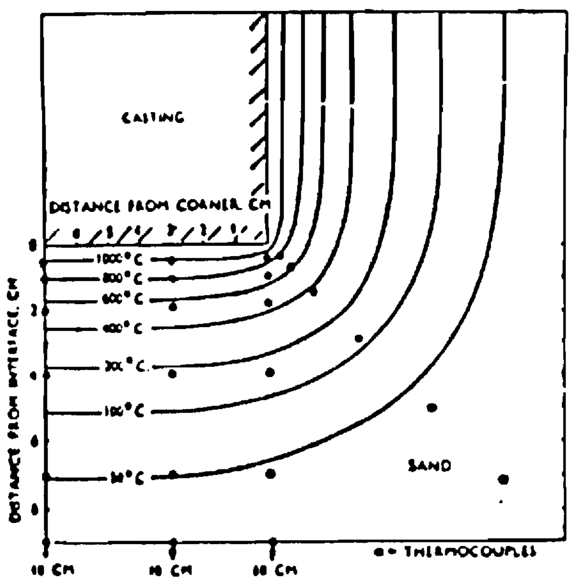

Figure 7. Isotherms at sharp 90^{0} corner, sand mold min. after pouring. Metal medium was cop base alloy.

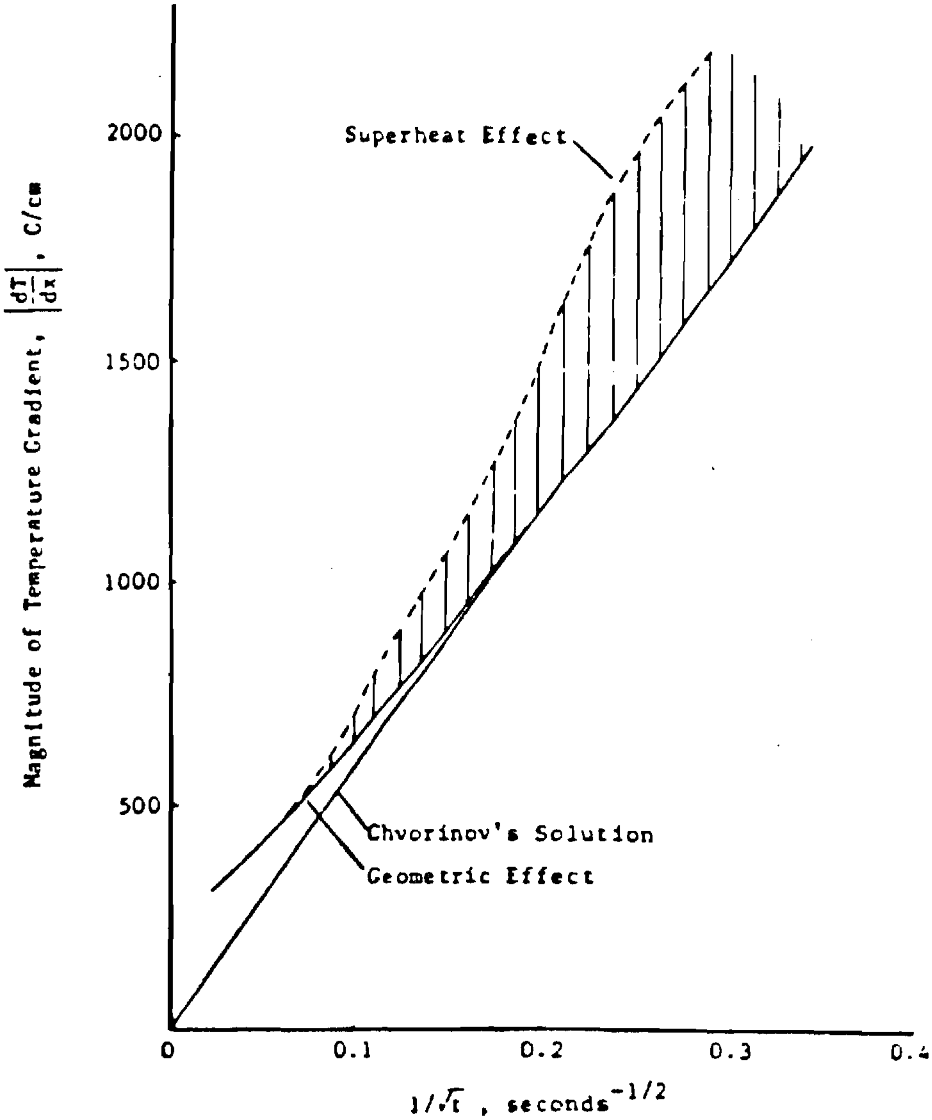

Figure 8. Interfacial temperature gradient for a 60° wedge showing typical edge and superheat effects (shematic).

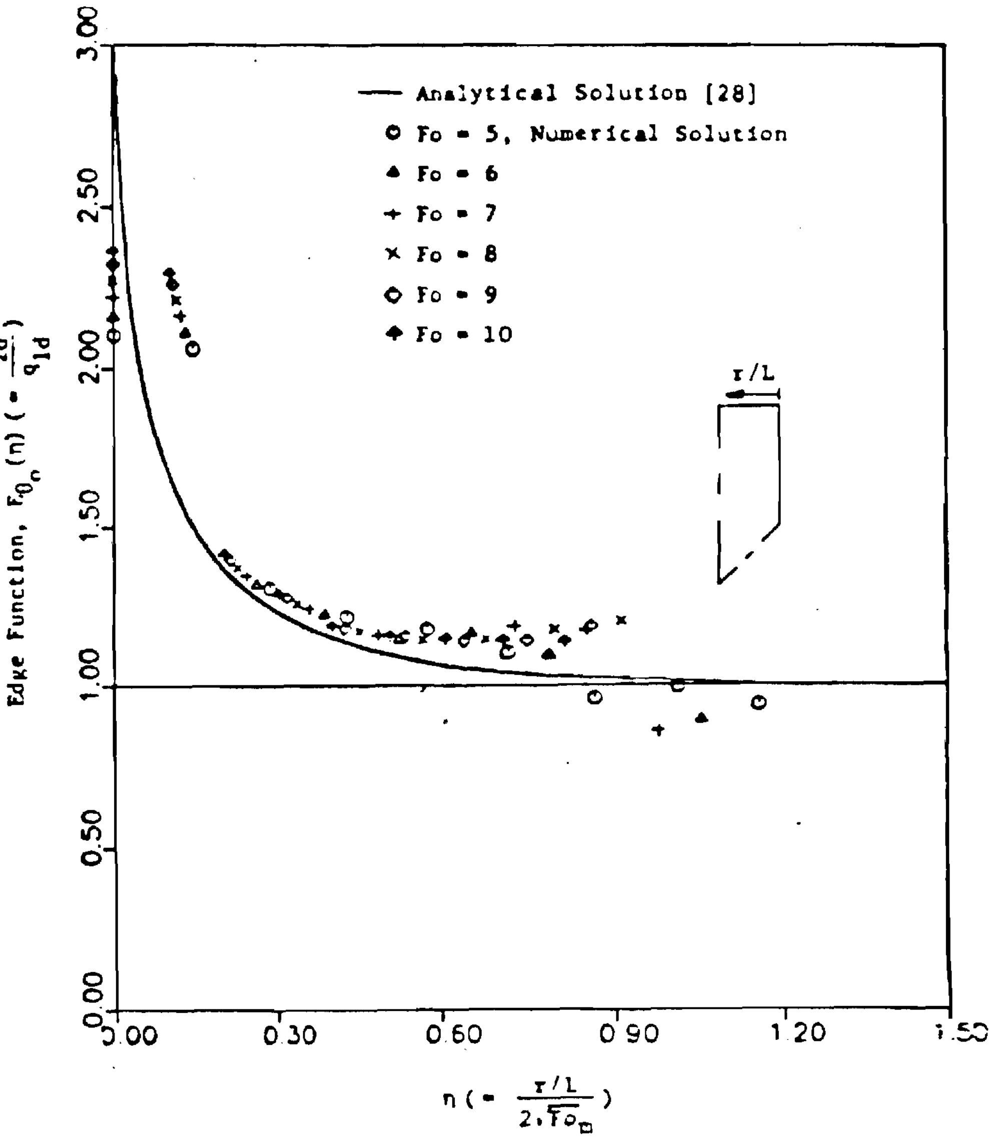

Figure 9. Computed edge function for cruciform (horizontal interface) with temperature dependent properties.

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APPENDIX A.I

COMPUTER AIDED DESIGN FOR CASTING AND SOLIDIFICATION TECHNOLOGY CADCAST

by

John T. Berry* and Robert D. Pehlke+

ABSTRACT

The National Science Foundation has been supporting a large-scale project of a multidisciplinary nature at the Georgia Institute of Technology and the University of Michigan, which is concerned with the geometric (solid) modelling of castings and their rigging and the simulation of the solidification sequence in the model. In parallel with this effort, data bases are being developed to describe mold and casting media thermal properties and how convective phenomena control initial temperature distributions. Work has recently commenced on modelling solidification in an investment cast plate.

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INTRODUCTION

The simulation of shaped casting solidification using the digital computer, probably goes back to at least the early nineteen-sixties when Forsund (1) in Denmark applied the technique to the problems of casting surface quality.

Prior to that time the work of Paschkis and co-workers using the analog computer ranges back to the nineteen-forties, when work sponsored by the AFS Heat Transfer Committee at Columbia University revealed many hitherto unknown or only partly anticipated aspects of casting solidification (2).

Three reviews of the various historical aspects of simulation should be referred to. Firstly, the AFS monograph by one of the present writers (RDP) and his colleagues - describing background and later work, again sponsored by the AFS Heat Transfer Committee at the University of Michigan (3). Secondly: a review prepared by Durham and Berry (4) which discusses broad aspects of casting solidification simulation. Thirdly; a review by Erickson (5) which contains an excellent bibliography to 1980.

In addition to these papers the publication lists of the CADCAST project* and the proceedings of the two Engineering Foundation Conferences of Casting and Welding Modeling, published by ASME, are also appropriate (6).

BACKGROUND TO CADCAST PROJECT

In 1979, the National Science Foundation funded a project to be conducted jointly at the University of Michigan and the Georgia Institute of Technology, which concerned itself with some of the various roadblocks which were then judged as being critical in the more widespread application of shaped casting simulation by computer:

The Georgia Institute of Technology's portion of the program was directed to:

(a) Evaluating the current state of the art of of computer-aided geometric modeling, in particular the interface with computation schemes (TASK I).

(b) Building appropriate models of thermal transport in the molding medium, using both theoretical and experimental approaches (TASK II).

(c) Determining the extent of thermal convection in the liquid metal within the mold cavity, using both computational and experimental means (TASK IV).

* See Bibliography

The University of Michigan's portion of the program was directed to:

(a) Evaluation of the overall and relative costs of various computer-based methods of simulation, including finite element versus finite difference techniques (TASK III).

(b) Investigating and quantifying the nature of heat transfer across interfaces, including gaps which may form between a casting and the mold medium or chill (TASK V).

(c) Extending knowledge of how thermal expansion-contraction phenomena within the mold or the casting interact with gating and feeding in affecting unsoundness distribution (TASK VI).

The investigation has been guided by a steering committee representing all aspects of the industry, its technology-related trade associations, and the National Science Foundation. Liaison is also maintained with other interested U.S. governmental agencies. The investigation should prove to have a major impact upon both the productivity and product quality of the metalcasting industry.

RESEARCH PROGRAM TO DATE (a) TASK I

As will be seen the program has been an extremely interdisciplinary one involving a wide array of experimental and intellectual resources.

From the outset the program has stressed the important role of geometrical or solid modeling in the CAD system of the future. Consequently TASK I has involved a thorough examination of solid modelers in terms of their suitability in a metal casting solidification context.

Ideally at the center of any truly flexible CAD system should be a means of presenting and storing information related to the geometry of the casting and its rigging (Fig. 1). Although there are currently several turnkey CAD systems which can provide representations of three-dimensional objects, not all are based on true solid modelers. It is our view that casting and solidification dedicated CAD systems must be based on a truly three-dimensional geometric modeler. In such a system, all information related to topology and geometry-related features - surfaces, edges, vertices, through- and blind-holes, pockets, etc. - should be encoded and stored efficiently and unambiguously within the memory of the host computer.

In the present project, several currently available modelers have been examined for the compactness, ease of usage, capacity, flexibility, and general utility in foundry-related applications. Several test castings have been chosen for the purposes of this evaluation. Figure 2 shows a TIPS-1 based model of a test casting with its risers attached. The number of statements needed to build the model concerned was approximately twenty. Similar models have been built using either the ROMULUS, PADL-1 or CAE*PAC systems. The CAE*PAC system has some special advantages which make it particularly attractive to the foundryman. These advantages relate principally to the compactness of the system inasmuch as it can be supported on a small- or medium-sized computer; for example, IBM PC and VAX 11/780 systems are capable of handling most of the required modeling, viewing, sectioning and related operations.

One extremely important sector of activity in which further work is needed relates to the development of packages which will allow one to interface a modeler with the solver, or numerical computation routing.

Although there are now several such enmeshment packages which will subdivide the shape under consideration into elements and nodes, as well as construct files describing the nodes concerned, many are in the private domain and some quite expensive. The problems of enmeshment and file management have been described in a master's thesis form by CADCAST team member Dalton. The package described links together an earlier version of the CAE*PAC modeler with a general purpose 2-D finite element heat conduction solver FE-2D written by team member J. G. Hartley.

Currently available in the private domain are 3-D enmeshment packages such as SDRC's SUPERTAB routine which is linked with that organization's modeler GEOMOD. General Motors Corporation has also linked together their GM Solid to several computational routines through a package called SMUG. Most recently a new package linking the very compact CAE*PAC modeler to similar FEM computational codes has been announced. The package concerned is of particular interest since it will run on a small personal type computer. The computation portion would, of course, still be undertaken on a larger machine such as a VAX 11/780.

It should be added that this whole area is still undergoing important developments, although it is undoubtedly one of the most difficult interfaces one encounters in this particular area of software.

Thus, at the present time, although more work is needed to facilitate the development and assembly of sets of packages of the type mentioned above, the modelers alone are capable of providing the foundryman and patternmaker alike with information related to volume, area, and/or modulus properties both for rigging elements as well as for selected zones of castings. On the other hand, some commercially available pre-processors will permit the building of what is termed "wire frame" models, which can subsequently be enmeshed and then interfaced with various numerical solution routines. It must be emphasized, however, that the models so constructed are not true solid models and cannot provide any pictorial or section-related information.

(b) TASK II

The activity of TASK II has been concerned thus far with constructing a data base of thermal properties of mold materials utilized in conventional green and dry sand molding.

The transport of heat into the mold walls exerts a direct influence upon the profile of the solidification from within the casting. Although data relating to apparent thermal conductivity, specific heat, and ramming density of certain particulate mold materials have been available for a number of years, the very complex nature of the bodies concerned compounds the problem of property determination and property prediction.

The principal thrust of the experimental portion of our program in this area has been to document as accurately as possible the effects of some of the more important variables encountered in such materials. In particular, team member Hartley and co-workers have concentrated their research effort upon the effects of dry density, sand type, binder content, initial moisture, and temperature upon apparent thermal conductivity. Although most of the sand molding being done throughout the world is still undertaken using green-sand techniques, it was decided that before the heat-transport mechanism in the moist particulate could be elucidated, the nature of the phenomenon in dried sand compacts should first be fully understood. It should be noted that for many thicker sectioned castings (>50mm), especially those cast in high-melting point materials (>1300K), the overall solidification behavior does not differ greatly between dry-sand practice and green-sand practice.

Initially, the team concentrated upon measurements of the influence of initial moisture content, binder content, and temperature on the apparent thermal conductivity of bentonite bonded silica and zircon sands. The technique used for the measurements was the thermal probe or single thermocouple method. The range of temperature over which measurements have been made so far has been from room temperature up to 1023 K. A successful comparison of the results obtained with those of previous workers has also been undertaken.

In an effort to rationalize these observations, as well as to lay the groundwork for a predictive model of thermal conductivity of dry sand, team member Hartley and colleagues have made considerable progress towards obtaining many of the necessary components of the model that are not already available. This had included determination of the thermal conductivity of the bentonite bonding materials.

It is currently planned that the next phase of our investigations in this area will involve formulation of models for both green sand as well as investment media.

(c) TASK III

One of the important early activities of the University of Michigan team in TASK III was the examination of some of the currently available finite difference and finite element computational codes.

Three programs were selected for in-depth evaluation and testing during the project: ANSYS, MARC, and MITAS II. Two finite-element programs and one finite-difference program were represented; the finite-element programs are generally more convenient in terms of flexibility, whereas the finite-difference program is more efficient in terms of computing time.

All three programs - NASYS, MARC, and MITAS II - were eventually used successfully to make transient heat-conduction calculations in one-, two-, and three-speace dimensions.

A great deal of time was spent learning how to run these programs, particularly with remote computing sites, although the personnel at Swanson Analysis, MARC, and Martin Marietta were extremely cooperative and helpful. Time did not permit actual metal-casting simulations to be run; many more weeks of full-time work would be necessary to do so.

Table 1 compares the three programs against the ideal goal of a heat-transfer simulation capability for metal castings. Individual running costs are interpreted as being low, medium, or high if they are expected to be below \$100, in the range \$100-\$1,000, or above \$1,000 per run, respectively. In summary, either ANSYS or MARC can be recommended for the simulation of heat transfer in metal castings if the foundry is prepared to:

(1) Spend at least \$1,000 per simulation

(2) Train at least one person to work full-time on running these programs, anticipating that an induction period of a few months would be needed before any really productive achievements would materialize.

MITAS-II can also be recommended for the simulation of heat transfer in metal castings if the foundry is prepared to:

(1) Spend at least a few hundred dollars per simulation

(2) Compensate for the relative lack of pre- and post-processing capabilities by making an extra effort in the preparation of input data and in interpreting the program output

(3) Train at least one person to work full-time on running these programs, anticipating that an induction period of a few months would be needed before

any really productive achievements would materialize.

The cost of making several runs with any of the above programs could easily equal the cost of purchasing a large microcomputer or a small minicomputer - that is, in the \$10,000 to \$40,000 range. An attractive alternative for the foundry industry therefore would be to:

(1) Develop a nationally available general-purpose, heat-transfer code that is dedicated to the simulation of heat transfer in metal castings, and which can be run on a small (\$20,000-\$40,000) minicomputer.

(2) Have each foundry purchase such a minicomputer and operate it locally, rather than attempt to run a program at a remote computing site. In this way, there would be almost unbounded freedom in making several numerical simulations of potential castings. The cost of the computer could pay for itself within some 20, 30, or 40 simulations. The computer would probably also be available for other computing activities within the foundry.

(3) Base the code on the finite-element method because of its higher ability to handle unusual geometries. (A finite-difference code, however, would be a reasonable alternative if the finite-element program proved to be somewhat too expensive to run.)

Following this evaluation it was felt that some of the disadvantages exhibited by the large scale general purpose codes necessitated the construction of special purpose software.

The University of Michigan has thus developed a new compact finite-element simulator specifically for the modelling of casting solidification. Key features include direct accounting for latent heat, temperature dependent material properties, and the ability to accept arbitrary geometries. A full description of this simulator will be available later from Beffel, Wilkes, and Pehlke.

Currently, test simulation of a variety of cast shapes is being performed. One that is of particular interest to this Institute is of an investment plate casting that was poured at the Materials and Manufacturing Technology Center of TRW. Experimental thermocouple data were also provided by TRW under the supervision of Michael Robinson.

The investment plate casting measures 6.75 tall by 1.25 inches wide. There were three different thicknesses of plates, 0.125, 0.250, and 0.375 inches wide. The 0.250 inch thick plate was chosen for simulation. The actual casting consisted of a spoke arrangement of 15 plates protruding from a central hub riser. IN100 was the cast metal used, and solidification was carried out in an evacuated furnace to guarantee that no oxidation of the liquid metal took place.

A photograph of the casting is shown in Figure 3. Material properties are temperature dependent and were furnished by TRW.

The IN100 was super-heated to 2650°F and the mold was pre-heated in an evacuated furnace to 1900°F. All thermocouples were monitored every five seconds until all of them had dropped below 1900°F.

The finite-element simulation used a model of the upper one-half of the plate and the upper riser. The model was symmetric in the Z-direction, and so only one side of the plate was modelled. This finite-element model used 984 elements and 1294 nodes. The three different material types used in the simulation were the metal, the mold, and a modified metal used to simulate the extra heat flow from the central hub riser to the top riser. Radiation boundary conditions were used on all exposed surfaces, and insulated boundary conditions were imposed on the surfaces created by the symmetry of the model. Time steps of 36 seconds in length were used and the simulation was run until 20 time steps had passed, or until 720 simulated seconds had elapsed.

The results of an initial simulation are shown in Figure 4. The solid line is the experimentally measured temperature at the center of the 0.250 inch thick plate. The points are the two finite-element nodes adjacent to the thermocouple location.

While these results are preliminary, it should be noted that the predicted time of solidification is within ten percent of the experimentally measured solidification time, and the subsequent cooling curve is closely matched by the calculations. Additional simulations are being performed for comparison with other experimental data.

(d) OTHER TASKS

Another important activity of the University of Michigan has been an exhaustive search of the literature describing the thermal properties of various cast alloys.

It was seen from this review that for most alloys the thermal conductivity in the liquid state and the specific heat were not reported. The thermal conductivities for the liquid state were estimated for these alloys by assuming that values of two factors, viz, the rate of change of thermal conductivity with temperature in the liquid state, and the ratio of the thermal conductivity at the solidus temperature to that at the liquidus temperature, were equal to those of the pure base-metal. The specific heats were estimated at several temperatures by taking the weighted averages of the specific heats of the individual elements in that alloy.

The temperature dependence of thermal conductivity and specific heat are represented by straight line or parabolic segments. The expressions for these segments were derived through regression analysis of the experimental or estimated values. A booklet, now available, contains the derived expressions for thermal conductivity specific heat, and density for each alloy and mold material; for each alloy, the latent heat of fusion, and solidus and liquidus temperatures are also given. This booklet should contribute to the advancement of computer-aided design of castings.

Other aspects of the accurate simulation of the freezing of shaped casting for example, interfacial contact with chills, convective effects during and after pouring and volumetric shrinkage and mold wall movement effects have also been studied as part of the program.

All of these aspects have been reported on in the progress reports of the two CADCAST teams as well as in the open literature items which are appended to this paper. Many of the findings described should prove to be of interest to investment casting organisations.

SUMMARY

A large-scale investigation on computer-aided design of castings has been conducted by collaboration at two major academic institutions. The areas selected for study at the University of Michigan are:

- Computation costs and alternative simulation methods
- Interfacial boundary conditions
- Thermal gradients, volumetric shrinkage, and mold wall movement
- The thermal properties of metals and alloys

At the Georgia Institute of Technology, the following areas selected for study are:

- Geometric modeling-numerical simulation interface
- The thermal transport model for the mold
- The filling transient model - involving both (a) free and (b) forced convection

The program has been interactive between the participants and has been guided by a committee representing the casting industry. An industry/university cooperative program to demonstrate computer-aided design of castings is planned. As we enter the third phase of our program, this phase also involves experiments and computation with investment molds and castings.

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Feature	Ideal	ANSYS	MARC	MITAS-II
Ability to learn how to run it on a remote computer	Easy	Difficult	Moderately difficult	Moderately difficult
Running cost	Low	High	High	Moderate
Ability to account directly for latent heat	Yes	No	Yes	Yes
Dedicated heat transfer code	Yes	No	No	Yes
Accuracy	Good	Very good	Very good	Very good
Pre- and post-processing capabilities	Good	Good	Good	Poor

Table 1. A Comparison of ANSYS, MARC, and MITAS-II with the Ideal Goal for a Metalcasting Simulation Capability

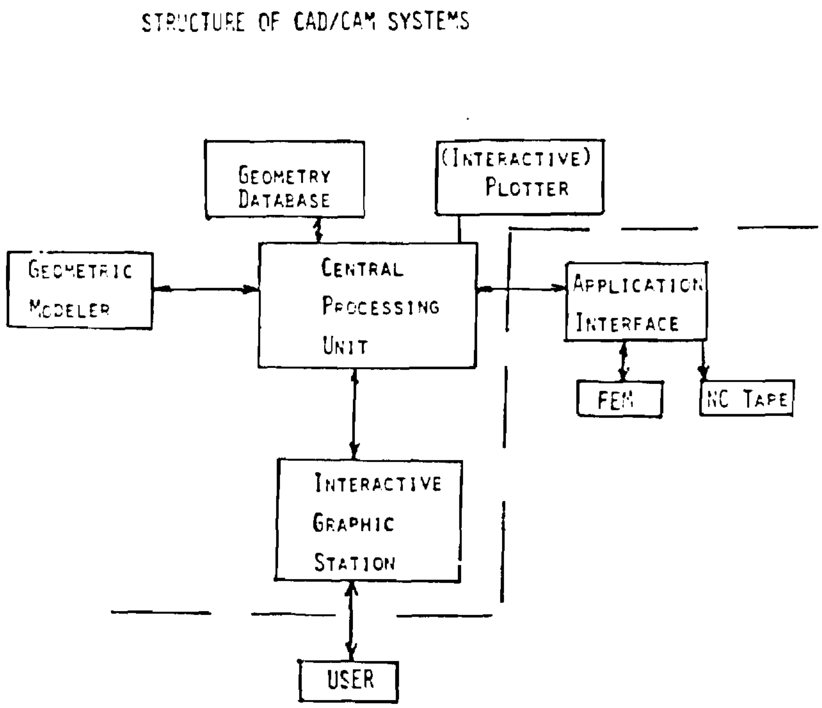

Figure 1. Schematic representation of CAD system for casting application

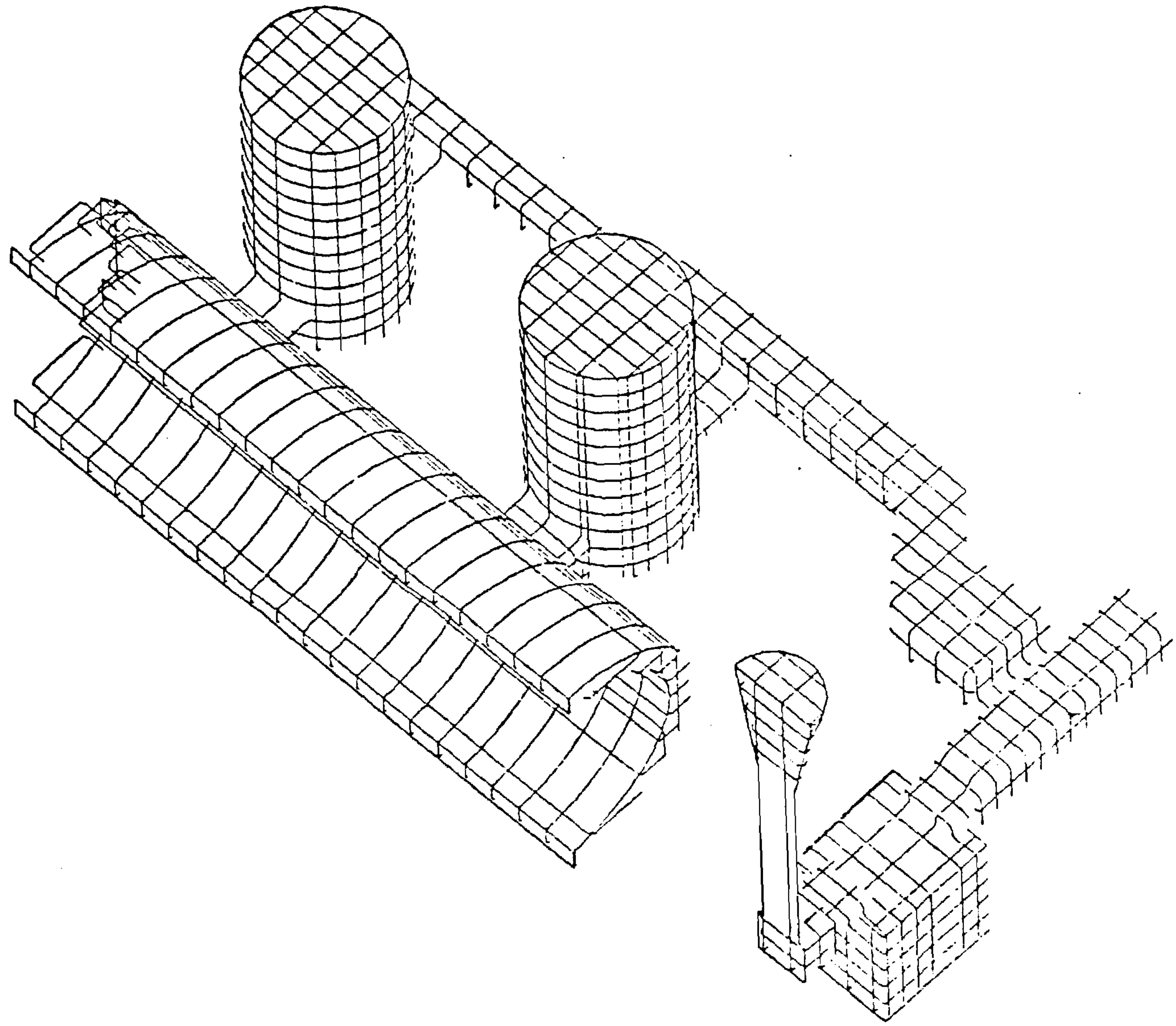

Figure 2. Machinability Test Piece with Commercial Foundry Rigging, Modeled Using TIPS-1.

Figure 3. Plate Cluster Casting

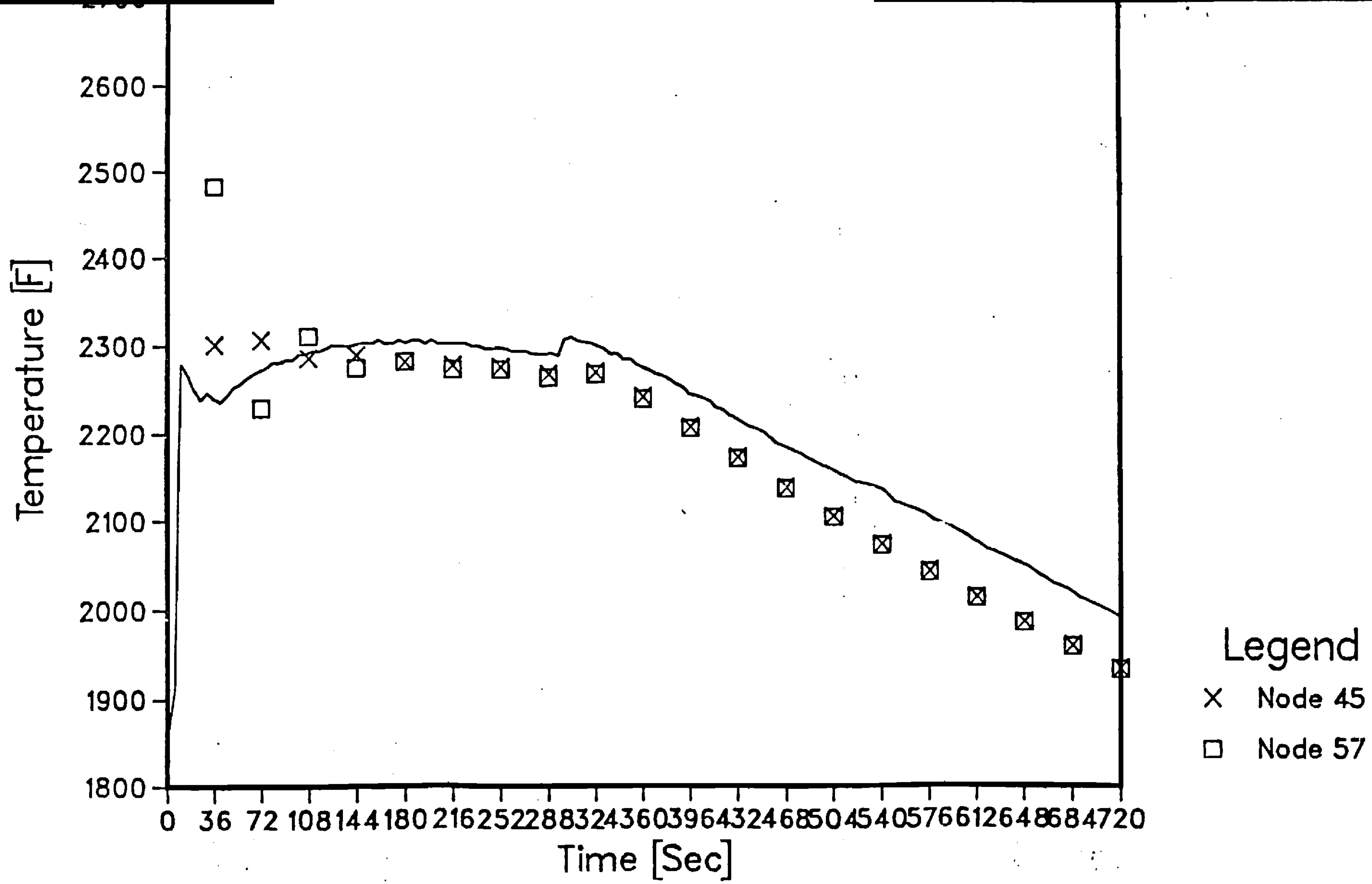

Figure 4. Preliminary Simulation Results for Plate Cluster Casting
Center of Plate Experimental Versus Numerical Results

APPENDIX A.II

CALCULATION OF FEEDING RANGE DATA FOR THE HYPO-EUTECTIC A-357 ALLOY USING THE RESULTS OF AN FEM SOLIDIFICATION MODEL

by

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Submitted for Publication

Transaction American Foundrymen Society

1985

ABSTRACT

The presence or absence of unsoundness due to solidification shrinkage is dependent upon the range of action of risers in sand castings. As established by Pellini and co-workers [1,2,3], the feeding range of a riser for a given alloy is an empirical estimate of this range of action. This measure depends upon many factors including casting geometry, the thermal properties of the mold and casting materials and the mode of solidification of the cast alloy. In recent years a great deal of work has focused upon modeling the solidification of sand castings using numerical solutions to the non-linear heat conduction problem [4,5,6,7,8]. These models are capable of providing information about temperature fields. Davies [9] has used information derived from finite difference (FDM) models in determining feeding ranges for various cast alloys using a simple fluid flow model which includes an empirical calibration based upon the data of Pellini et al. [1,2,3].

In this study a finite element (FEM) formulation of the heat conduction problem is incorporated to solve for the temperature field in a simple plate casting of A357 alloy in silica sand. Information derived from the numerical solution is then used to estimate feeding range for the alloy in the manner outlined by Davies [9]. The results are compared with those reported by Davies [9] for Al 7% Si alloy and are discussed in terms of their implication of a distribution of dispersed shrinkage along the length of the plate casting.

INTRODUCTION

It is a well established principle, perhaps due to the pioneering work of Pellini and his co-workers [3], that the presence or absence of unsoundness due to solidification shrinkage is not only dependent upon the freezing order of joined sections but also upon the range of action of the risers in sand castings. Although dealing primarily with short freezing range alloys (SFRA), Pellini et al. established the concept of a feeding range attributable to a given alloy. Specifically, they defined the feeding range of a riser for a given alloy as the distance over which a riser could supply feed metal, producing a sound cross-section, in plate and bar castings. This distance was given as a multiple of the plate or bar thickness. Unsoundness in the SFRA is, primarily, the presence of centerline shrinkage. In long freezing range alloys (LFRA), unsoundness is primarily dispersed interdendritic porosity. In LFRA feeding of solidification shrinkage occurs via "mass feeding" and interdendritic fluid flow with the former term distinguishing the bulk flow of liquid plus solid crystals. This is because solidification occurs in a "pasty" manner with the entire casting or major parts of it consisting of a mixture of solid and liquid phases. In the SFRA, on the other hand, solidification proceeds via growth of the solid from the mold walls with a more or less distinguishable boundary between liquid and solid phases. The end result is that the feeding of solidification proceeds by virtue of capillary flow through the resulting channel. The differences between the modes of feeding and resulting forms of unsoundness in SFRA and LFRA are illustrated schematically in Figure 1. Only a limited amount of work has been done to characterize feeding ranges for LFRA. No empirical data similar to that of Pellini et al. are available for the many alloys of commercial importance falling into this category.

Piwonka and Flemings [10] have shown that flow through "mushy" zones obeys relations similar to those employed in flow through other types of porous beds. However, data applicable to feeding ranges of risers was not reported. Davies [9] has calculated feeding ranges for several alloys in plate castings by employing the velocity of the 80% solidified front isotherm calculated from a finite difference formulation of the differential enthalpy equation in a simple model of fluid flow. The aspects of Davies' analysis important to the problem studied here are as follows: for laminar flow through a tube of radius, r, and length, ℓ , the pressure drop is

$$\Delta P = \frac{8\mu\ell\dot{Q}}{\Pi r^4} \tag{1}$$

where μ is the viscosity of the fluid, and $\dot{Q}$ is the volumetric flow rate. By conservation of mass, the flow rate required to feed solidification shrinkage is

$$\dot{Q} = V_s \pi r^2 m \tag{2}$$

where m is the solidification contraction and V_s is the solidus velocity. Equating (1) and (2) and solving for ℓ yields the capillary feeding distance, since the flow rate $\dot{Q}$ is just the amount necessary to feed shrinkage. Thus,

$$\ell = \ell_f = \frac{\Delta P r^2}{8\mu m V_s} \tag{3}$$

where ℓ_f is the feeding distance. The result is highly approximate at best for metals solidifying with a plane front. Davies seeks to correct Equation (3) to incorporate the effects of a not-so-clearly defined solidification front by means of an empirical correction factor, B. Since we can write $\Delta P = \gamma P_0$ where P_0 is the pressure due to metallostatic head and γ a constant, Equation (3) can be rewritten as

$$\ell_f = \frac{BP_0 r^2}{\mu m V_s} = \frac{B' h \rho_\ell r^2}{\mu m V_s} \tag{4}$$

where h is the height of the metallostatic head and ρ_ℓ is the density of the liquid metal. Davies then chooses B' such that a maximum feeding distance of 4.5 S results, where S is the plate thickness. In doing so, he employs data involving 0.6%C steel cast in silica sand. Since this empirical value (4.5 S) is generally accepted for this combination of alloy and mold material, this step is probably permissable. The resulting model is then used to calculate feeding ranges for some pure metals, eutectics and SFRA by employing solidus velocities obtained from FDM solutions to the differential enthalpy equation.

To calculate feeding ranges for LFRA, Davies uses the same basic model with a modified approach which attempts to incorporate observed differences in the nature of feeding in these types of alloys. During the first part of solidification mass feeding (movement of liquid and solidified crystals) takes place until a solid dendrite network prevents this from occurring. This is generally believed to occur at approximately 70 percent solidified. When mass feeding stops, feeding will continue via interdendritic fluid flow. To obtain some estimate of the length of interdendritic feeding distance, Davies uses a simple idealization of the dendrite network geometry to calculate an effective channel radius to be used in Equation (4). He then finds the mean velocity of the 80 percent solid front from the FDM solution and substitutes this for V_S to obtain ℓ_f as the interdendritic feeding distance. The 80 percent solid front velocity is chosen as it is observed that feeding terminates near that point in the solidification (80 percent), perhaps due to surface tension and other effects which are difficult to analyze.

Davies has calculated feeding range data for Al 7% Si alloy which is close in composition to alloy A357 (Al 7% Si 0.6% Mg), having the same hypoeutectic character which gives a mixed dendritic/eutectic solidification mode. He found the end contribution to be 5.5 S and the feeder contribution to be 4.5 S yielding a maximum feeding distance of 10 S, for the 7 percent silicon alloy. It must be noted here that Davies treated the Al 7% Si alloy as an SFRA. Thus, the value of feeding range was that estimated to be required to prevent centerline shrinkage. He notes in his discussion that plates cast in Al 7% Si which were ten times the thickness in length (equal to maximum feeding range) showed no centerline shrinkage but <u>did</u> show some microporosity. Davies also calculated interdendritic feeding distance for Al 2% Si alloy (an LFRA) and estimated percent microporosity from his data. He found that in simulations without an end chill, percent microshrinkage varied somewhat along the plate but was relatively constant at 1.5 percent. When a simulation was of a casting with an end chill, however, he found a lower average percent porosity which increased from zero percent at the chilled end to about 1.5 percent at the feeder end. His results, for the unchilled plate, however, agreed qualitatively with experimental results.

Numerical Analysis

A numerical analysis of the heat transfer problem with the dimensions of a plate casting as shown in Figure 2 using liquidus and solidus temperatures for A-357 alloy has been performed.

A finite element model has been developed which can simulate the solidification of sand castings assuming uniform initial temperatures in the casting and mold. The model can take into account temperature dependent thermal properties of the mold and cast material and will allow latent heat generation at constant temperature or over a temperature range. These characteristics make the model suited for studying the solidification of the plate casting considered in this analysis. Specifically, a model for the solidification of the hypoeutectic A-357 alloy has been formulated for this study.

The model involves a finite element formulation of the differential enthalpy equation first solved using finite differences by Sarjant and Slack [11] and used extensively by Hansen [12,13] in solidification studies. The finite difference formulation has been shown by Meyer [14], via a rigorous mathematical proof to converge to the weak solution of the Stefan problem as the time step and node spacing approach zero. A summary of the formulation as used in this study follows. A more complete description has been given [15].

For a metal at rest with density ρ specific heat C, and thermal conductivity k, conservation of energy can be written

$$\frac{\partial E}{\partial t} = \nabla . k \nabla T \tag{5}$$

where T is temperature, t is time and

$$E = \int_0^{T-T_s} (\rho C + \rho\lambda\delta)dT \qquad (6)$$

In Equation (6) T_s is the liquidus temperature and λ is the latent heat of fusion with δ producing a discontinuity of strength λ in the function E at $T = T_s$. A modified temperature scale is then defined as

$$\phi = \frac{C_s}{\lambda} \int_0^{T-T_s} \frac{k}{k_0} dT \qquad (7)$$

where C_s is the specific heat at $T = T_s$ and k_0 is the thermal conductivity at a reference temperature. Equation (5) can then be written as

$$\frac{\partial H}{\partial Fo} = \nabla^{*2}\phi \qquad (8)$$

$$H = \frac{1}{\lambda} \int_0^{T-T_s} (C + \lambda\delta)dT \ , \qquad (9)$$

∇^{*2} is the dimensionless Laplacian operator and $Fo = k_0 t/\rho C_s L^2$. For the mold material

$$\frac{\rho_m}{\rho} \frac{\partial H_m}{\partial Fo} = \frac{k_{om}}{k_0} \nabla^{*2} \phi_m \qquad (10)$$

where the subscript m denotes mold material and

$$H_m = \int_0^{T-T_s} \frac{C_m}{\lambda} dT \qquad (11)$$

$$\phi_m = \frac{C_s}{\lambda} \int_0^{T-T_s} \frac{k_m}{k_{om}} \, dT \qquad (12)$$

Equations (8) and (10) are the governing field equations for the mold and solidifying metal subject to the initial conditions

$$H = H_o \text{ in } \Omega \qquad (13)$$

and boundary conditions

$$H = H_{S_1} \text{ on } S_1 \qquad (14)$$

$$\nabla^* \phi \cdot \vec{n} + \overline{Bi} \, \frac{C_s}{\lambda} \, (\phi - \phi_\infty) = 0 \text{ on } S_2 \qquad (15)$$

where

$$\overline{Bi} = Bi \, (T - T_\infty)/(\phi - \phi_\infty) \qquad (16)$$

$$Bi = \frac{hL}{k_o} \qquad (17)$$

h is the heat transfer coefficient acting at S_2. In Equations (13) through (17) the appropriate properties are chosen depending upon whether the boundary lies on the mold or metal.

In order to model the hypoeutectic character of A-357 alloy (latent heat generation over a temperature range followed by generation at a constant temperature), Equation (9) can be modified as

$$H = \int_0^{T-T_e} \left(\frac{C}{\lambda} + \delta' + \frac{\partial f}{\partial T}\right) dT \tag{18}$$

where T_e is the eutectic temperature. The fraction solidified, f, during crystallization of the primary phase is determined by assuming equilibrium conditions and using the lever rule

$$f = \frac{T_s - T}{(T_m - T)(1 - \gamma)} \tag{19}$$

where γ is the equilibrium partition coefficient, T_s is the liquidus temperature and T_m is the melting temperature of the pure metal. It follows from Equation (18) that H = 0 at $T = T_e$. Letting H = 1 at $T = T_s$, it follows that H = 1 - f during primary solidification. Latent heat generation at the eutectic temperature is accounted for by choosing δ' such that $\delta' = 1 - f_e$ at $T = T_e$ where f_e is the fraction solidified from Equation (19) at $T = T_e$. Thus, a one to one correspondence exists between H and ϕ for the alloy except at the eutectic point where $0 < H < 1 - f_e$.

The finite element equations are derived using the standard Galerkin method procedure [16] with the interpolation functions being the same for H as for ϕ . This procedure yields the following matrix equations:

$$-[[K_{vm}] + [K_{s_{2m}}]] \{\phi_m\} = [C_m] \{\dot{H}_m\} - \{R_{s_{2m}}\} \tag{20}$$

$$-[[K_v] + [K_{s_2}]]\ \{\phi\} = [C]\ \{\dot{H}\} - \{R_{s_2}\} \tag{21}$$

The element matrices are

$$[K_v]^{(e)} = \int_{V^{(e)}} \nabla^*[N]^T . \nabla^*[N] dV^{(e)} \tag{22}$$

$$[K_{vm}]^{(e)} = \frac{k_{om}}{k_o} \int_{V^{(e)}} \nabla^*[N]^T . \nabla^*[N] dV^{(e)} \tag{23}$$

$$[K_{s_2}]^{(e)} = \frac{C_s}{\lambda} \overline{Bi} \int_{S_2(e)} [N]^T [N] dS_2^{(e)} \tag{24}$$

$$[K_{s_{2m}}]^{(e)} = \frac{C_s}{\lambda} \overline{Bi}_m \int_{S_2(e)} [N]^T [N] dS_2^{(e)} \tag{25}$$

$$[C]^{(e)} = \int_{V^{(e)}} [N]^T [N] dV^{(e)} \tag{26}$$

$$[C_m]^{(e)} = \frac{\rho_m}{\rho} \int_{V^{(e)}} [N]^T [N] dV^{(e)} \tag{27}$$

$$\{R_{s_2}\}^{(e)} = \frac{C_s}{\lambda} \overline{Bi} \int_{S_2^{(e)}} [N]^T \phi_\infty dS_2^{(e)} \tag{28}$$

$$\{R_{s_{2m}}\}^{(e)} = \frac{C_s}{\lambda} \overline{Bi}_m \int_{S_2^{(e)}} [N]^T \phi_\infty dS_2^{(e)} \quad . \tag{29}$$

In Equations (20) through (29), $\{\phi\}$ and $\{\dot{H}\}$ are the column vectors of nodal modified temperatures and time derivative of nodal dimensionless enthalpies respectively. The matrices [N] are the interpolation functions and $V^{(e)}$ denote the volume and S_2 surface of the element respectively. The time derivative may be replaced by

$$\{\dot{H}\}_\beta = \frac{\{H\}^{n+1} - \{H\}^n}{\Delta Fo} \tag{30}$$

and the nodal modified temperatures evaluated at time level $n\Delta Fo + \beta\Delta Fo$ by

$$\{\phi\}_\beta = \beta\{\phi\}^{n+1} + (1 - \beta)\ \{\phi\}^n \tag{31}$$

The system of equations to be solved at each time step is then

$$\frac{\{C\}}{\Delta Fo}\{H\}^{n+1} = \frac{\{C\}}{\Delta Fo}\{H\}^n - \beta\ [K]\ \{\phi\}^{n+1} - (1 - \beta)\ [K]\ \{\phi\}^n + \{R_{S_2}\}_\beta\ . \tag{32}$$

For $\beta = 0$ the algorithm is fully explicit and the $\{\phi\}^n$ can be determined from the $\{H\}^n$. For $0 < \beta < 1$ iteration is required within a time step since the $\{\phi\}^{n+1}$ are not known until the $\{H\}^{n+1}$ are calculated. In this study $\beta = 1/2$ and a "lumped" capacitance procedure was used. Mass lumping is a common practice and has been discussed in the literature [17,18,19]. The properties and parameters used in this study to model A-357 alloy solidification are listed in Table 1. The model phase diagram derived from these parameters (nominal 7% Si) is shown in Figure 3. The applicable dimensionless enthalpy modified temperature relationship [Equations (C-18) and (C-19)] is shown in Figure 4. Symmetry about the centerline of the plate was assumed in order to reduce computational effort. This is not expected to introduce significant error except perhaps in regions near the riser. Insulated boundary conditions were applied at the outer mold boundaries. This was found to be acceptable since semi-infinite conditions were maintained for the solidification time involved. The metal was assumed to be initially at the liquidus temperature and the mold was initially at room temperature. Constant properties as listed in Table 1 were assumed for both the metal and mold materials. Latent heat generation was assumed to be distributed evenly over the entire solidification period so that the fraction of latent heat liberated during primary solidification was equal to the fraction solidified during that period and the remainder of the latent heat was released at the eutectic temperature.

Analysis and Results

The location along the center line of the plate of various fraction solidified fronts versus time is shown in Figure 5. From this diagram it can be seen that at approximately Fo = 0.027 (t = 4 min) after the start of solidification, a large region along the centerline of the plate is nearly solidified. The capillary length between a given fraction solid curve chosen and the 100% solid curve is the vertical distance between these curves. Capillary length is plotted in Figure 6 versus position along the centerline of the plate for various fraction solid curves. The capillary feeding distance, ℓ_f , can be calculated using Equation (4) and the properties or parameters listed in Table 2 along with the velocity of the 100% solid front, V_s. The latter velocity is calculated from the numerical solution. The feeding distance ℓ_f , versus distance along the centerline of the plate is also shown in Figure 6. The limits of centerline shrinkage are then the points of intersection of the curves for capillary length and the capillary feeding distance, ℓ_f. The feeding ranges, assuming feeding problems starting at various fractions solidified, are given in Table 3 as multiples of the plate thickness, S/L. These results are in close agreement with those obtained by Davies [9] for Al 7% Si alloy.

Because of the hypoeutectic character of A-357 alloy it is felt that it is not meaningful to treat the alloy in a manner similar to the way in which Davies [9] treated Al 2% Si alloy (having the character of an LFRA) because eutectic solidification begins at 62.5% solid in the model of A-357 alloy whereas dendritic growth continues up to some much larger fraction solid in Al 2% Si. However some qualitative assessment of the variation in microporosity along the plate length can be made if we simply assert that feeding through the solid dendrite network becomes more difficult as the solid fraction increases. Thus, the amount of microporosity will depend upon the imbalance between the ability to feed (volumetric flow rate of liquid metal through the network) and the tendency to consume (velocities of the various fraction solid fronts). Because of the gradual increase in the rate of movement of the 70% through 100% solid fronts followed by very rapid movement in the center of the plate followed by a gradual decrease, the interdendritic feeding distance will gradually decrease, then rapidly decrease, then gradually increase. This should lead to a microporosity profile which gradually increases, rapidly reaches a plateau at the middle of the plate and then falls off rapidly near the riser end. Thus a uniform distribution of porosity should be obtained in the center of the plate.

Conclusions and Observations

1. The results of the analysis carried out in this study are in good agreement with the results of Davies [9].
2. Approximate, quantitative assessment of design parameters to prevent gross (centerline) shrinkage porosity for the alloy modeled in this study (A-357 alloy) and other alloys of commercial importance can be derived from temperature field data obtained from a numerical solution to the solidification problem.
3. Qualitative assessment of the distribution of microporosity (interdendritic shrinkage) can be obtained for a particular casting geometry from the temperature field data.

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NOMENCLATURE

Bi	Biot number, hL/k_o
$\overline{Bi}$	modified Biot number, Bi $(T - T_\infty)/(\phi - \phi_\infty)$
B'	empirical correction factor
C	heat capacity of cast material
C_s	heat capacity of cast material at liquidus temperature
E	enthalpy
f	fraction solidified
Fo	Fourier number of cast material $k_o t/\rho C_S L^2$
ΔFo	dimensionless time step size
h	metallostatic heat height in Equation (C-4), heat transfer coefficient in Equation (C-17)
H	dimensionless enthalpy of cast material
H_o	initial dimensionless enthalpy of cast material
k	thermal conductivity of cast material
k_o	reference thermal conductivity of cast material
L	characteristic length (length of mold)
m	solidification contraction
$\vec{n}$	unit normal vector to surface
ℓ	length of capillary
ΔP	pressure difference
P_o	metallostatic pressure
$\dot{Q}$	volumetric flow rate
r	capillary radius
S	plate thickness
S_1	specified temperature boundary surface
S_2	specified convection boundary surface
t	time

T	temperature
T_e	eutectic temperature
T_m	melting point of pure metal
T_s	liquidus temperature
T_∞	ambient temperature
V_s	velocity of 100% solid front
x'	distance along centerline measured from end of plate
β	fraction of a time step at which FEM equations are solved
γ	equilibrium partition coefficient for Si in Al
δ	Dirac delta function evaluated at liquidus temperature
δ'	delta function of strength $1 - f_e$ evaluated at the eutectic temperature
λ	latent heat of fusion of cast material
μ	liquid metal viscosity
ρ	density of cast material
ρ_ℓ	density of liquid metal
ϕ	modified dimensionless temperature of cast material
Ω	solution domain

Subscripts

m	denotes mold material
e	denotes eutectic

Superscripts

n	time step
(e)	denotes element
T	denotes transpose of matrix

Matrices

$[K_v]$	volume integral of stiffness matrix

$[K_{S_2}]$ surface integral of stiffness matrix

$[C]$ capacitance matrix

$\{H\}$ dimensionless nodal enthalpy vector

$\{\dot{H}\}$ time derivative of nodal dimensionless enthalpy vector

$\{\phi\}$ dimensionless nodal modified temperature vector

$[N]$ interpolation functions

$\{R_{S_2}\}$ vector due to surface integral

FIGURE CAPTIONS

Figure 1	Solidification Pattern at a Given Time, Schematically, a) Pure Metals; b) Alloy with a Wide Solidification Range [9]
Figure 2	2-D Finite Element mesh Used in the Numerical Analysis
Figure 3	Model Phase Diagram Derived from the Parameters Listed in Table C-1 and Used in the Numerical Analysis
Figure 4	Dimensionless Enthalpy vs. Modified Temperature for A-357 Alloy
Figure 5	Location Along the Centerline of the Plate of Various Fraction Solid Fronts Versus Time
Figure 6	Capillary Length and Capillary Feeding Distance Versus Distance Along the Plate Centerline

Table 1

Properties and Parameters Used in the Numerical Analysis

$\rho = 2.67\ g/cm^3$	ref [9]
$\rho_m = 1.50\ g/cm^3$	ref [9]
$C = C_s = 1.05\ J/g^oC$	ref [9]
$C_m = 1.12\ J/g^oC$ (Silica Sand)	ref [9]
$k = k_o = 1.87\ W/cm^oC$	ref [9]
$k_m = k_{om} = 0.0084\ W/cm^oC$ (Silica Sand)	ref [9]
$\lambda = 432.6\ J/g$	ref [9]
$\gamma = 0.13$	ref [20]
$T_s = 613^oC$	ref [21]
$T_e = 557^oC$	ref [21]
$T_m = 660^oC$	ref [21]
$L = 76.2\ cm$	

Table 2

Properties and Parameters Used to Calculate ℓ_f from Equation (C-4)

$B' = 0.22$	ref [9]
$\rho = 2.3\ g/cm^3$	ref [9]
$r = 0.75 \times 10^{-2}$ cm	ref [9]
$\mu = 0.030$ poise	ref [9]
$m = 0.05$	ref [9]
$h = 14.1875$ cm	

Table 3

Feeding Ranges for Feeding Problems Starting at Various Fractions Solid for A-357 Alloy

Fraction Solid	Feeding Range		
	end	feeder	max
90%	6.75 (S/L)	3.75 (S/L)	10.5 (S/L)
80%	5.2 (S/L)	3.3 (S/L)	8.5 (S/L)
70%	4.4 (S/L)	3.0 (S/L)	7.4 (S/L)
Eutectic	0 (S/L)	2.2 (S/L)	2.2 (S/L)

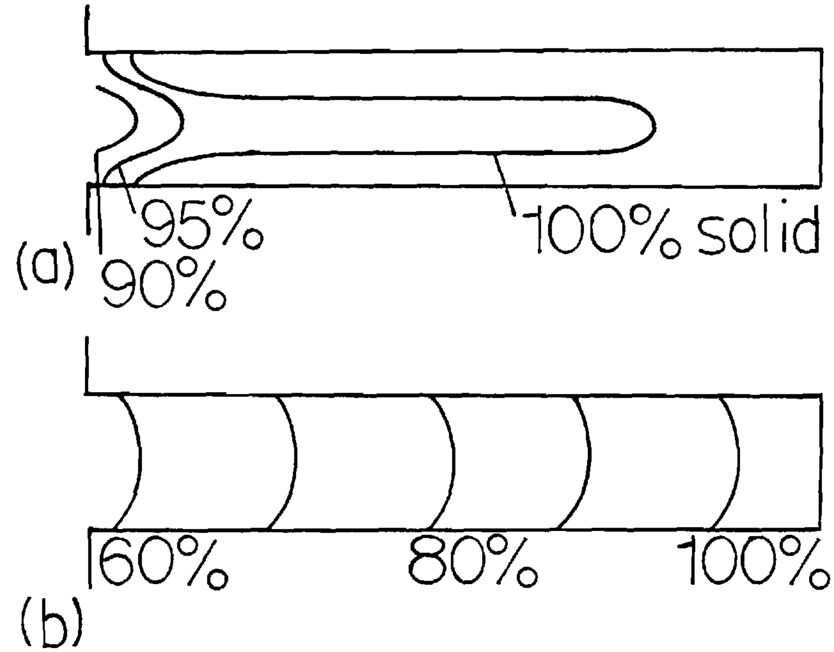


Figure 1.

Solidification Pattern at a Given Time, Schematically,
a) Pure Metals; b) Alloy with a Wide Solidification Range [9]

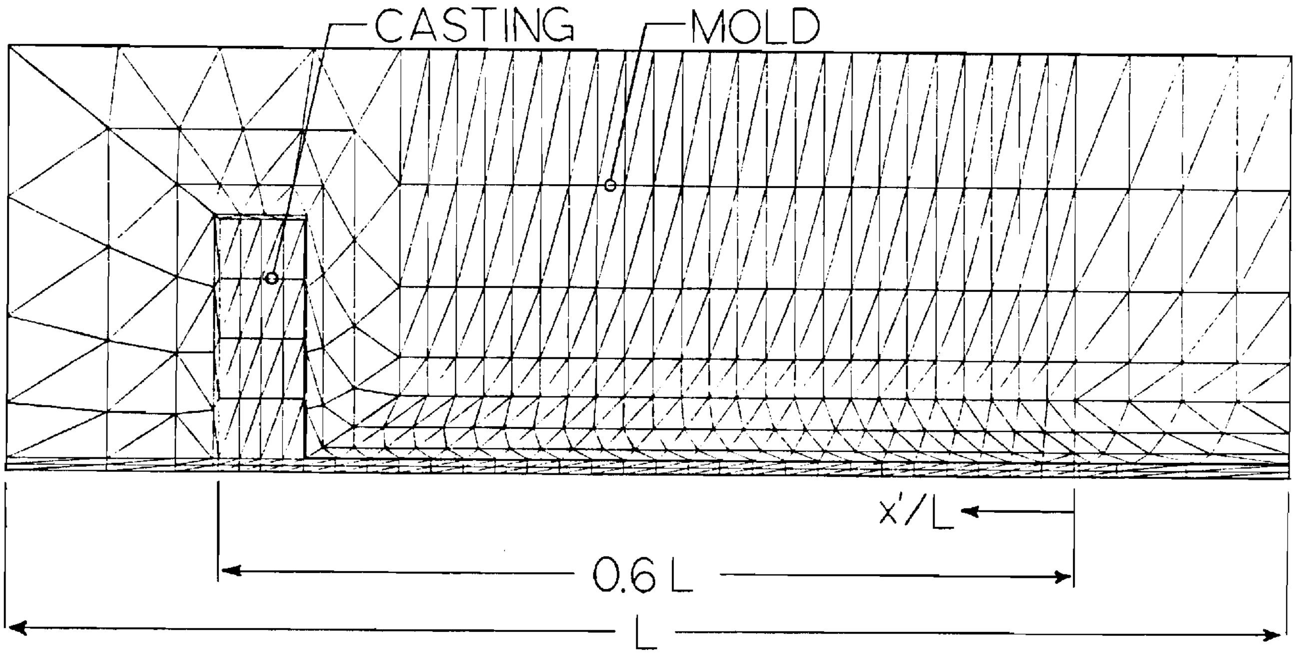
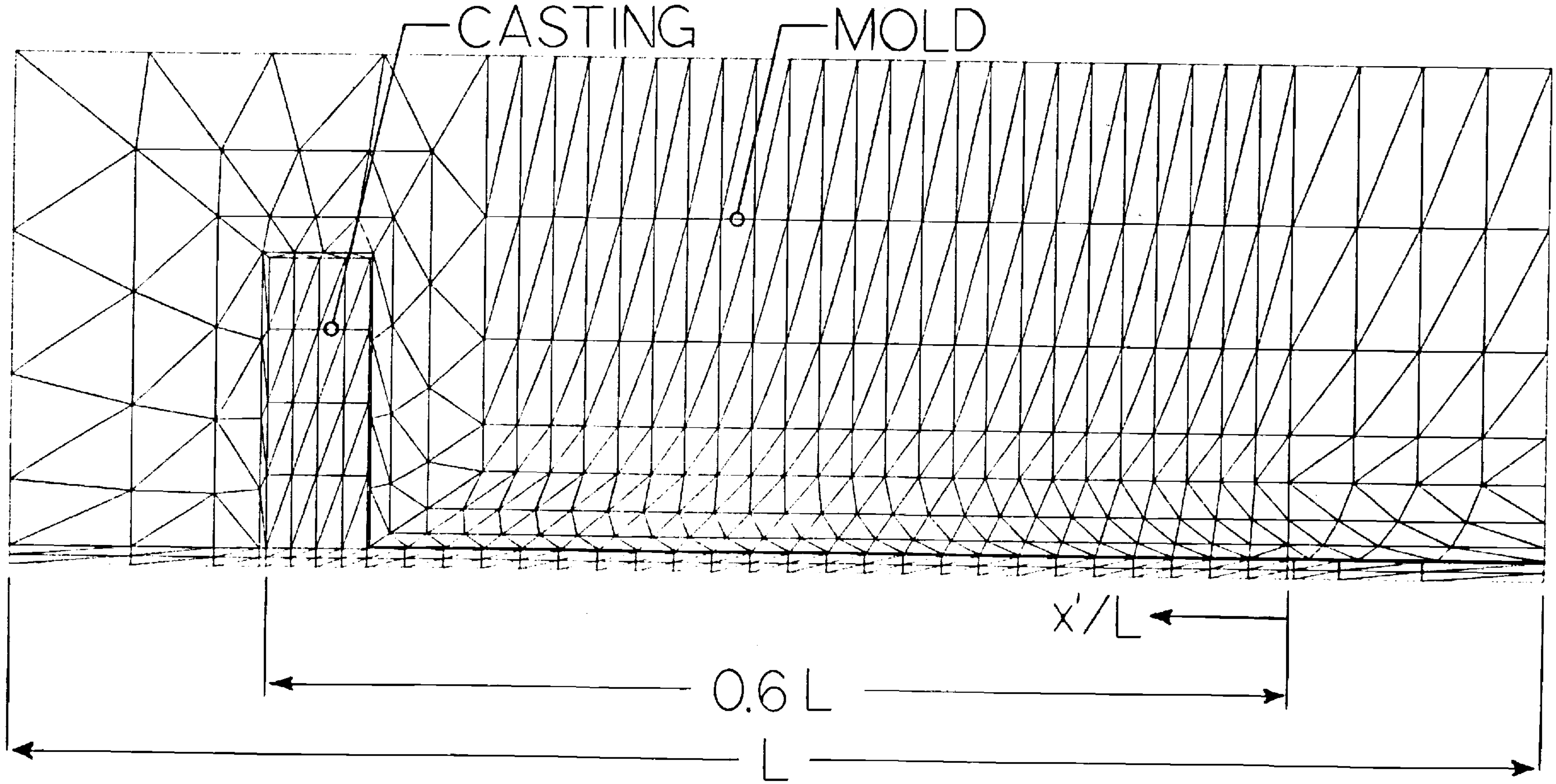


Figure 2

2-D Finite Element Mesh Used in the Numerical Analysis


CASTING
MOLD
X'/L
0.6 L
L

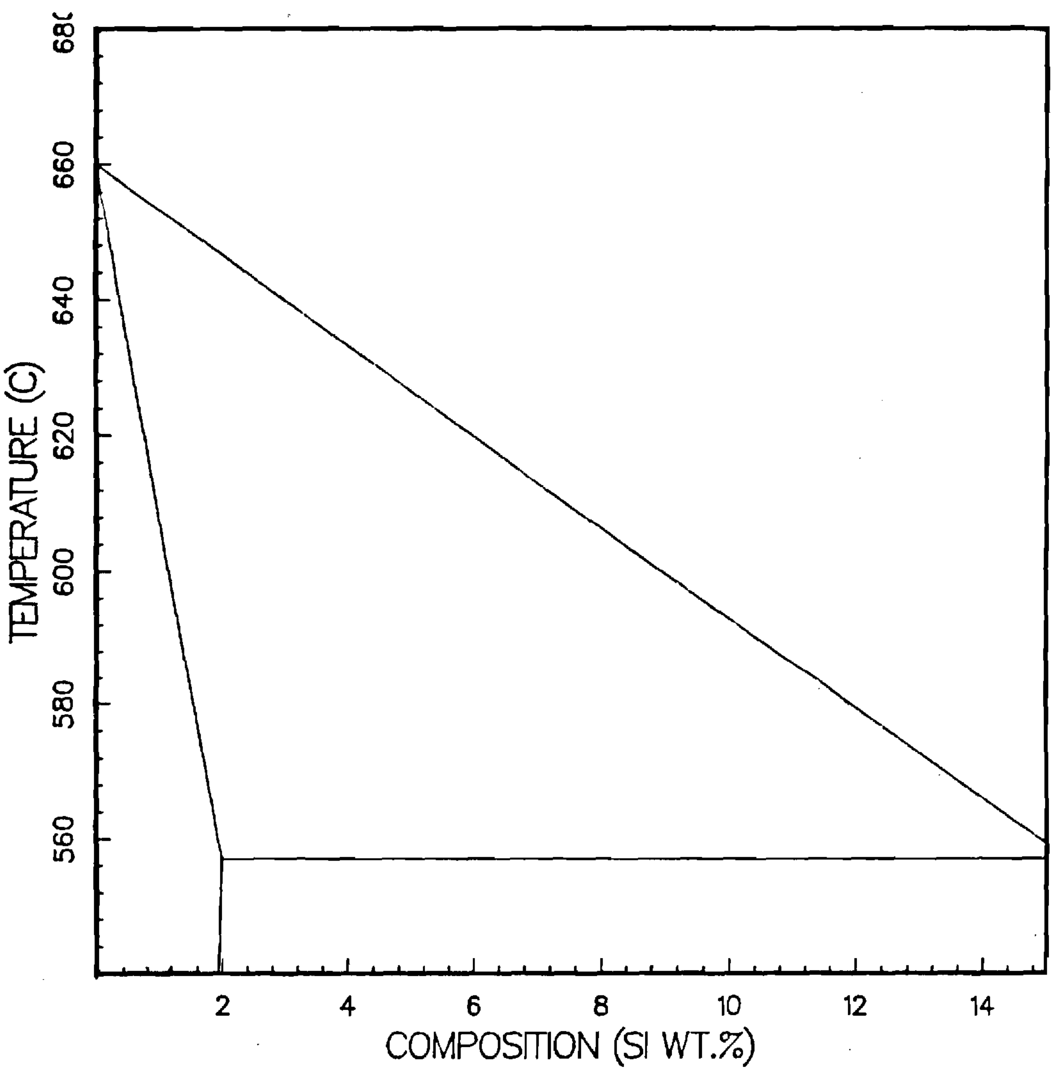


Figure 3

Model Phase Diagram Derived from the Parameters Listed in Table C-1 and Used in the Numerical Analysis

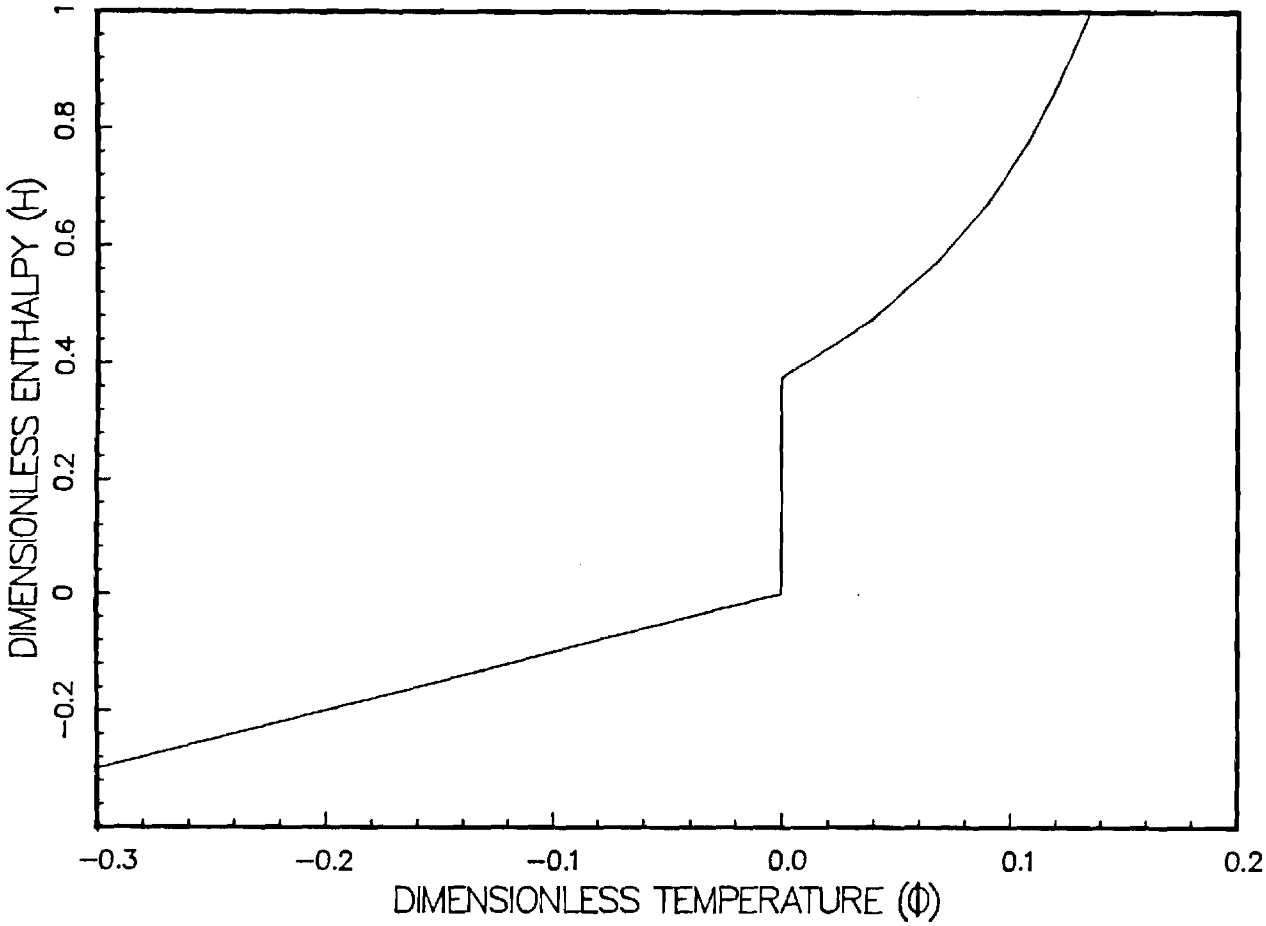


Figure 4

Dimensionless Enthalpy vs. Modified Temperature for A-357 Alloy

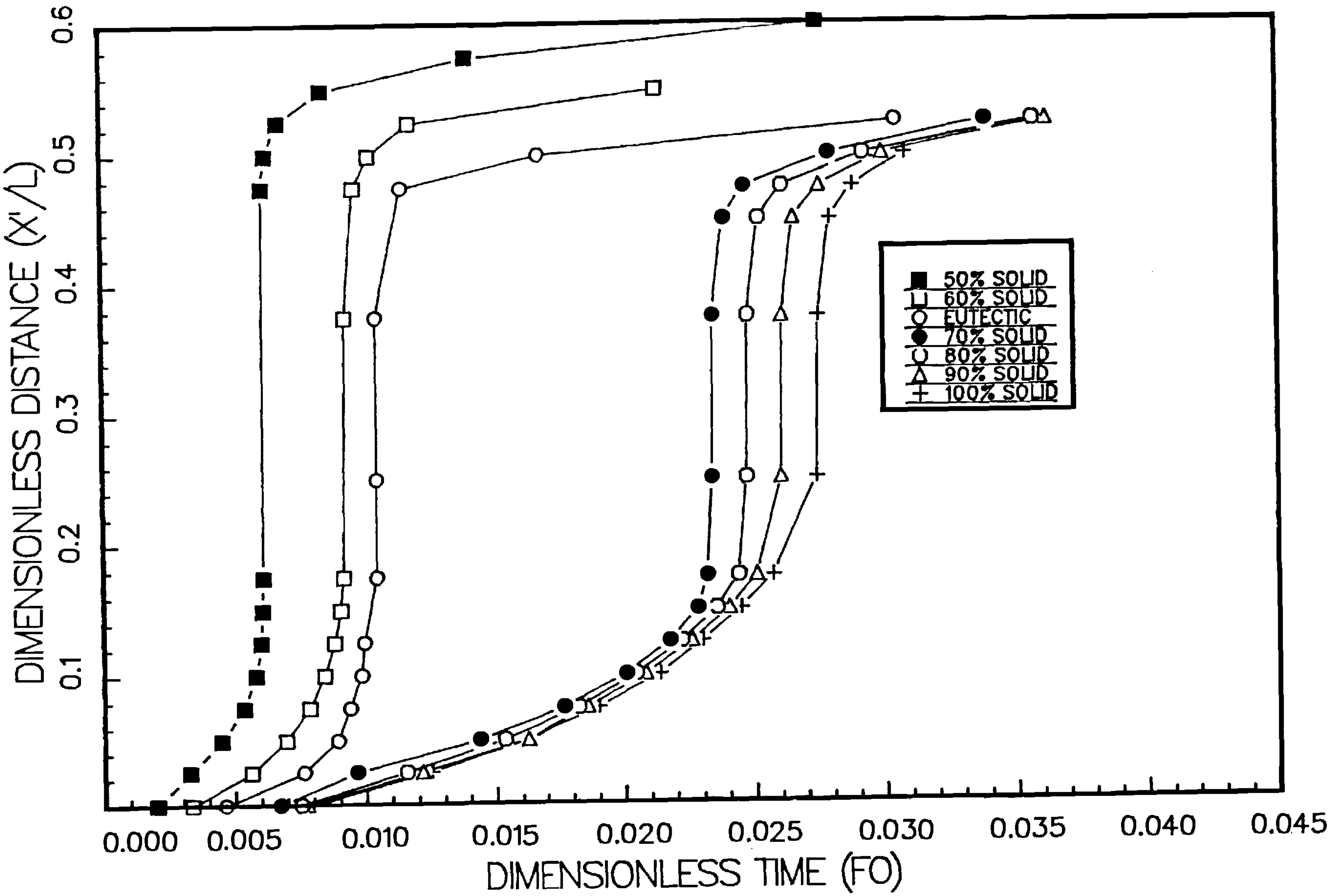


Figure 5

Location Along the Centerline of the Plate of
Various Fraction Solid Fronts Versus Time

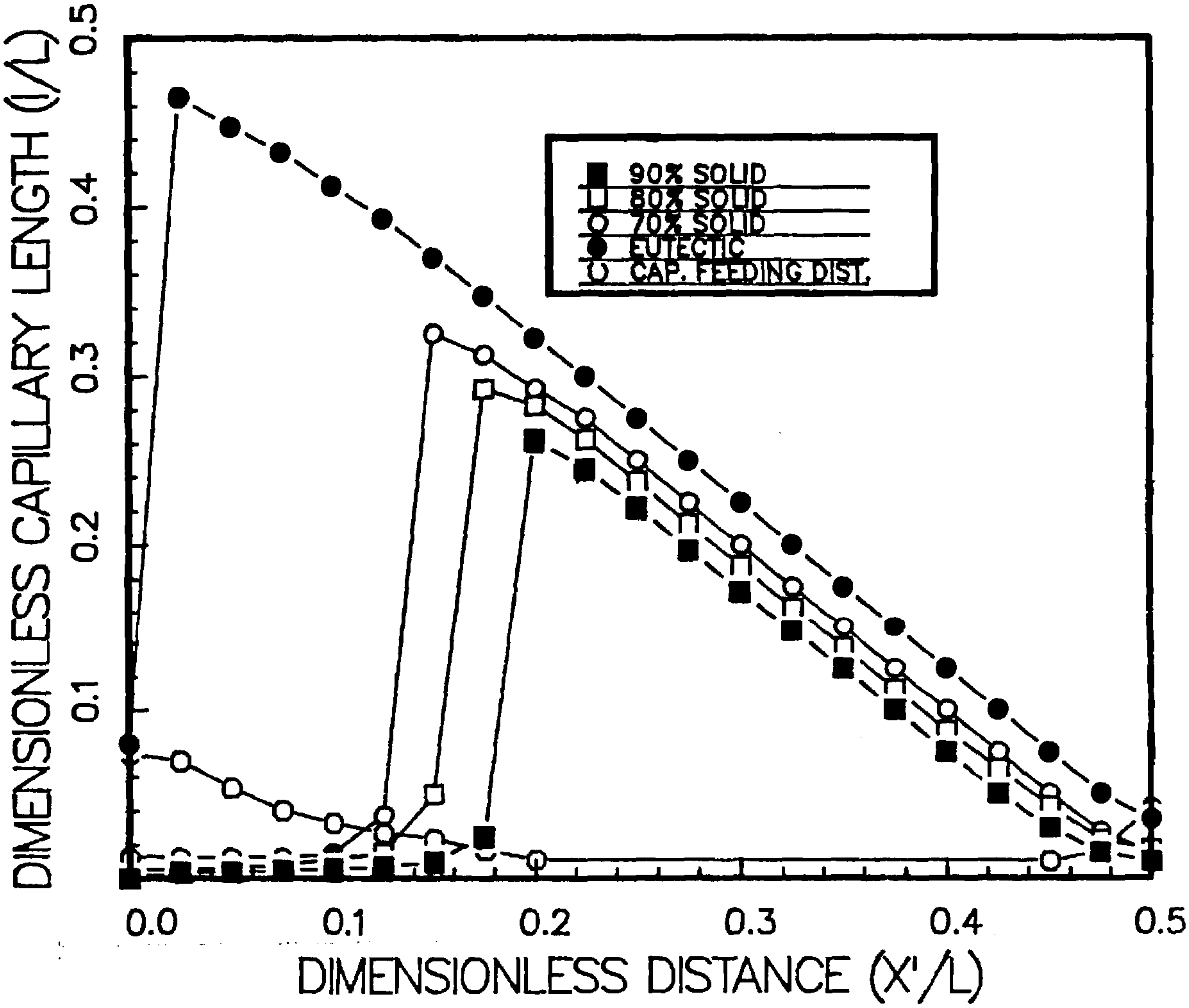


Figure 6

Capillary Length and Capillary Feeding Distance
Versus Distance Along the Plate Centerline

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I—PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Woodruff School of Mechanical Engr. Georgia Institute of Technology Atlanta, Georgia 30332	Div. & Design, Manufacturing & Computer Engr.	MEA 82-11524
	4. Award Period From 4/1/83 To 9/30/86	5. Cumulative Award Amount $484,461.00

6. Project Title

A Computer-Aided Design System for Castings

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

The project has continued the examination of problems associated with the computer aided design and engineering of castings. The research undertaken has involved three universities: The Georgia Institute of Technology and The University of Michigan throughout the grants' three years and additionally the University of Alabama (under sub-contract) during the last year. The task areas examined by Georgia Tech and the University of Alabama have been concerned with the geometric representation of the casting and rigging and its interface with the computational system, the exploration of the material property data base, the modeling of free and forced convection effects during and immediately after pouring, and finally the control and prescription of interface heat flux in casting.

In the course of the investigation the following has been accomplished:

(i) Three geometric modelers have been evaluated for casting/rigging representation
(ii) A survey of geometric modelers has been undertaken (in collaboration with, the VTT, Technical Research Center of Finland)
(iii) The steps required to link successfully a geometric model to a two-dimensional FEM transient heat conduction program have been enumerated and test examples run
(iv) A theoretical model for predicting the thermal conductivity of dried sands has been developed successfully
(v) A theoretical model for predicting the thermal conductivity of green sands has been formulated
(vi) An experimental method for validating the above model has been developed (in collaboration with VTT)
(vii) A comprehensive model for describing temperature history during the filling of both vertical and horizontal portions of the gating system has been formulated
(viii) The model for describing temperature history so developed has been partially validated in collaboration with experimenters at the Technical University of Denmark (DTH)
(ix) The use of heat pipes to control interfacial heat flux has been investigated using a finite difference model which is linked to a heat pipe simulation, wherein operating parameters can be manipulated
(x) A compact method of describing the naturally occuring (non-controlled) heat flux at the mold-metal interface in simple two dimensional rectangular or polygonal mold enclosures has been further developed and applied to riser dimensioning

The results of the above research have been disseminated in some thirty-five publications, four Ph.D. studies and eight masters theses. In collaboration with representatives of the University of Michigan team, presentations of the results to workshops convened by national and international bodies have been undertaken.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	Approx. Date
a. Abstracts of Theses				X	
b. Publication Citations		X			
c. Data on Scientific Collaborators			X		
d. Information on Inventions	X				
e. Technical Description of Project and Results				X	
f. Other *(specify)*					

2. Principal Investigator/Project Director Name (Typed)	3. Principal Investigator/Project Director Signature	4. Date
4/83-4/86 John T. Berry * 4/80-9/86 Prateen V. Desai		26 June '86

NSF Form 98A (3-83) Supersedes All Previous Editions — Form Approved OMB No. 3145-0058

*Current address of former P.I.

Department of Metallurgical Engr.
University of Alabama
P. O. Box G., University, AL 35486

APPENDIX VII

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

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(See reverse side)

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM Check (√)	Approx. Date
a. Abstracts of Theses				✓	12-15-86
b. Publication Citations				✓	12-15-86
c. Data on Scientific Collaborators			X		
d. Information on Inventions	X				
e. Technical Description of Project and Results				✓	
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
4/86-9/86 Prateen V. Desai 4/83-4/86 John T. Berry		Oct 24, 1986

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